



An adaptive finite element method for the simulation of wood drying

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Abstract

Wood is a material with many industrial applications. In order to make it profitable, it usually must be subjected to a prior drying. When dealing with an uncommon wooden topology, the drying time is difficult to predict and so a numerical simulation is the best option. In this work, we present a numerical method for this purpose, with the advantage of an adaptive mesh based on an a posteriori error indicator.

1 Introduction

Wood is adopting an increasingly important role in many areas of science [4]. Wooden houses, bridges and all kind of structures are very common nowadays. In order to use wood in those applications, it is typically a prior requirement the drying of wood up to a compatible humidity level. As a consequence, drying technology is of fundamental importance and is being continuously developed.

One of the major drawbacks when evaluating drying technology is the testing phase previous to developing the equipment, which can require expensive trials, materials and facilities. One effective solution to overcome this disadvantage, consists of resorting to numerical simulations. Regarding wood drying, the amount of existing specialist software is scarce, and have serious limitations, since it does not take into account the intricate wood topologies where porosity can undergo abrupt changes and large anisotropy is present [5, 6, 7]. These features make the standard finite element method, which is probably the most popular, not suitable to give accurate results, since the finite element meshes should be refined over those regions where the abrupt changes occur. To overcome this issue, an adaptive finite element method based on an a posteriori error indicator can be applied.

In this work, we apply an adaptive finite element method presented [2] to simulate the high temperature drying of wood stacks in presence of an abruptly changing porosity and simulate the drying of a wooden board.

2 Problem description and modeling

Wood is composed of different parts, being heartwood and sapwood the most relevant. Sapwood is much more permeable. It is also remarkable the possible presence of knots, aggregates

with very low permeability. The board represented in Figure 1 is composed of sapwood, that corresponds to the plain white area, and of a knotty cluster made of several individual knots, corresponding to the red hatched areas. This is a complex geometry that requires simulation in order to predict the drying rate.

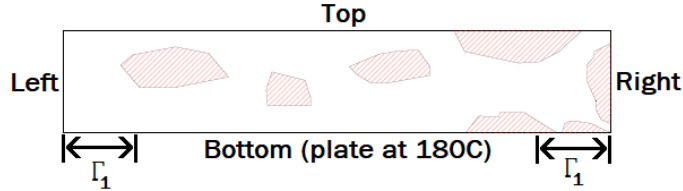


Figure 1: Domain of the wooden board, with sapwood (white area) and knots (in red).

Our goal is to carry out a 2D simulation of the drying at high temperature of the wooden board represented in Figure 1. The board dimensions are $0.5\text{ m} \times 0.09\text{ m}$, and the board is placed on top of a metallic plate at 180 C , temperature that makes water inside the boards to boil.

In order to model wood drying, we consider the Darcy-Forchheimer model, which is suitable when water velocity is moderate to high:

$$\begin{cases} \mu\mathcal{K}^{-1}\mathbf{u} + \beta\rho|\mathbf{u}|\mathbf{u} + \nabla p = \rho\mathbf{g}, & \nabla \cdot \mathbf{u} = f, & \text{in } \Omega, \\ \mathbf{u} \cdot \mathbf{n} = 0 & \text{on } \Gamma_1, & p = P_\Gamma & \text{on } \Gamma_2. \end{cases} \quad (1)$$

Here, Ω represents the wooden board. We denote by Γ the boundary of Ω and assume Γ is divided into two disjoint parts, Γ_1 and Γ_2 , with $|\Gamma_1|, |\Gamma_2| > 0$. In Figure 1, the boundary Γ_1 is indicated, and corresponds with two regions where the board is attached to the plate by screws; Γ_2 corresponds with the remaining boundary. The unknowns are the fluid velocity \mathbf{u} and the pressure p . The permeability tensor \mathcal{K} , which accounts for anisotropy, the fluid's dynamic viscosity μ , the Forchheimer parameter β , the density ρ , the gravity \mathbf{g} and f are known/data. Finally, we denote by \mathbf{n} the outward unit normal vector to Γ_1 . It is well known that under certain regularity assumptions on the data, problem (1) has a unique solution [1].

We remark that, since the Darcy-Forchheimer model (1) is stationary, our method is only suitable to predict what is called the *initial constant drying period*, which all kinds of wood exhibit. In practice, this limitation can be insignificant since in many scenarios the wanted humidity level is reached during the constant drying rate.

3 Numerical simulation and results

In order to solve problem (1), we implemented the adaptive finite element method presented in [2] in FreeFEM++ [3]. To refine a given mesh, we performed a loop: at each iteration, the error indicator estimates the error at each single element, and proceed to split those elements with the highest error.

Permeability of sapwood was set to $15,000\text{ m}^2$ and for the knots, 1 m^2 . At the bottom of the board a heating plate at 180 C is considered, resulting in a pressure of 2.1 bar at the bottom, whilst it was set to 1 bar at the top and both sides. As a result, water tend to exit the board mainly through the left side, since there are many knots blocking the right side. Water crossing the top is scarce, due to the high anisotropy of wood. This can be appreciated in Figure 2,

which shows both velocity and pressure gradient, and the adapted mesh. The computed drying rate is $0.237\%/min$.

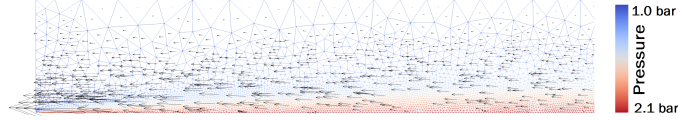


Figure 2: Adapted mesh, velocity vector and pressure

Finally, Figure 3 shows how the estimated error decreases with the increasing degrees of freedom (DOF) of the finite element method. It is clear that with an adaptive refinement, based on an a posteriori error indicator, the error is lower than with the usual uniform mesh refinement.

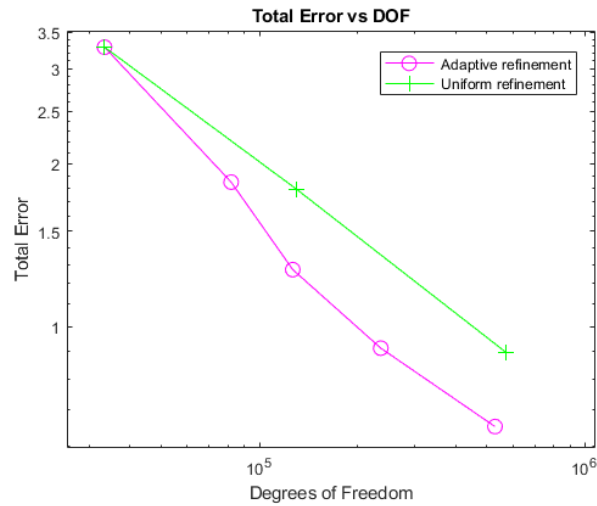


Figure 3: Evolution of error indicator with degrees of freedom

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