Rainfall frequency analysis using block maxima and peaks over threshold approaches.

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November 9, 2018

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Abstract In the present study, extreme rainfall frequency analysis was performed using block maxima (BM) and peaks over threshold (POT) approaches based on daily rainfall data gathered from three meteorological stations in Greece (Larisa, Aghialos, Trikala). In the first method, 9 different probability distributions (2 and 3 parameter distributions) were fitted to the samples (that were previously checked for randomness, trends and change points in the mean) in order to estimate rainfall depths for high return periods. According to diagnostic plots and Kolmogorov-Smirnov goodness of fit test, the fitting was acceptable for all the distributions that were considered in the three stations. However, the 3-parameter distributions and more specifically GEV for Larisa and Trikala and 3-parameter Lognormal for Aghialos, had a better fitting in the extreme values and could be considered more suitable for statistical modeling in these three stations. In the POT approach a suitable threshold was selected based on plots and statistical indexes. The excesses were modeled using the Generalized Pareto (GP) and a mixed model of Poisson and GP distributions. The estimated rainfall depths in this approach show similarities between the two models, as well as with those of the 3 parameter distributions used in the BM approach.

1 Introduction

Extreme rainfall is one of the main causes of natural disasters, especially in flood hazards worldwide. Not surprisingly, considerable attention has been paid in recent years to the modeling of extreme rainfall. An adequate knowledge of the frequency of recurrence of extreme rainfall is necessary for the proper design of structures such as dams, water supply and sewerage or irrigation networks. These structures are designed based on extreme rainfall events that correspond to high return periods.

Statistical frequency analysis is a valuable engineering tool when dealing with problems related to the frequency of occurrence of extreme events. Briefly, this methodology involves the extraction of samples of extreme rainfall events from a rainfall dataset and the fitting of theoretical probability distributions to the samples. The fitted distributions are used to estimate rainfall events for the desired return periods. Even though statistical modelling of extremes is widely used by the scientific community in various hydrological applications (Stedinger et al. 1993), there are still many tricky parts in the methodology that engineers and researchers need to deal with.

Firstly, defining the sample of the extremes can be a challenging task. There are two main approaches in the scientific literature for this purpose, namely, block maxima (BM) and peaks over threshold (POT) approaches. In the first method the sample is defined by the maximum value of the variable under study for a specific time period (usually a year) and in the second one by all the values that exceed a certain truncation level (Coles 2001, Villarini et al. 2011, Bezak et al. 2014). Difficulties in BM approach can be the relatively small resulting samples and the possibility of losing some extreme values. For example, an annual maximum precipitation value in a dry year may be lower than the second largest precipitation value in a wet year. On the other hand, major difficulties in the peaks over threshold approach are the selection of an appropriate threshold and assuring independence among the data values of the sample (Lang et al 1999, Engeland et al. 2004).

The next step is the selection of an appropriate probability distribution function that bests describes the random process. This task is challenging because usually more than one distribution may fit the data well (Villarini et al. 2011, Bezak et al. 2014). However, Generalized Extreme Value (GEV), Gumbel, 3 parameter lognormal, loglogistic, Pearson type III (Pea3) and Generalized Pareto (GP) are among the most commonly used distributions. Another, crucial choice in statistical modeling of extremes is the parameter estimation technique used for the fitting of the distributions. Maximum likelihood and L moments are among the most popular.

The main objective of this study is to estimate frequencies of recurrence of extreme rainfall events in Thessaly region in Greece. The two above-mentioned approaches were used in order to make comparisons. We fitted 9 different distributions (2 and 3 parameter distributions among them) in the BM and two in the POT approach to the samples of extreme rainfall in order to examine which of the distributions fit the data well and are more suitable for modeling the extreme values of rainfall in the three stations. Finally, we estimated rainfall depths using both approaches for various return periods.

2 Data and Methodology

2.1 Data

The analysis was based on daily precipitation data (units in mm) gathered from three meteorological stations located in Larisa, Aghialos and Trikala. The datasets were obtained from the database of the Hellenic National Meteorological Service and they cover a historical period from 1955-2004, 1956-2004 and 1974-2004 respectively.

2.2 Methodology

In the BM approach a block size is defined and the maximum (or minimum) events are selected for each block. In this study, Annual Maximum Series (AMS) were derived. The block size, time interval in this case, is a year and the sample is defined by selecting the maximum precipitation value for each year.

Statistical modeling of extreme events should be performed on samples that satisfy the assumption of randomness, no change points in the mean and no trends in order to assure that the results will be reliable. Therefore, the derived samples were checked for the validity of the above-mentioned assumptions using various statistical tests such as Bartel, Wallis & Moore and Wald-Wolfowitz for randomness, Buishand Range, Buishand U, Pettitt, Standard Normal Homogeneity and Lazante for change point detection and Mann Kendall and Cox & Stuart for trends. A detailed description of the above mentioned tests is provided by (Pohlert 2018) and references therein.

The null hypothesis (H_0) of the above mentioned tests are that the data are random and there are no trends or change points in the mean. The underlying assumption of each statistical test is evaluated on a specific significance level. In this study, the H_0 hypotheses were tested at the significance level a=0.05. This means that if the p value of the test is less than 0.05 then the H_0 is rejected. More details concerning the statistical tests can be found in the cited literature.

In this approach, two parameter (Gumbel, Exponential, Gamma, Lognormal and Logistic) and three parameter (Generalized Extreme Value (GEV), Lognormal, Loglogistic and Pearson type III) distributions were fitted to the samples of the stations. For further details one can refer to Coles (2001) and Reiss and Thomas (2007) who give a complete theoretical view and fairly accessible introductions to statistical analysis of extreme data.

In the POT approach, instead of defining a set of n random variables and the maximum of the set, a high enough limit u is defined and the extreme value analysis is performed using only the events which exceed this limit. The excesses were modeled using the Generalized Pareto distribution (GP) (Pickands 1975). In this study, we also fitted a Poisson-GP model in order to estimate the frequency of exceeding the threshold.

It should be noted that the limit u should be high enough to keep only the most extreme events in order for the estimations to be unbiased, but, at the same time, the number of the excesses should be sufficient in order to avoid uncertainties in the analysis. Therefore, the selection of the threshold is sensitive and for this reason it was based on propositions from the scientific literature (Groisman et al. 2005), plots, namely, mean residual life (MRL), parameters (shape and scale) and auto-tail dependence plots and the extremal index. Therefore, according to the various criteria, we allow the threshold to vary from station to station. According to the scientific literature, suitable thresholds are rainfall values that correspond to 95, 99 and 99.5% percentiles. Considering the MRL and parameters plots, suitable thresholds are identified in the area where there is an evident linearity in the plots. The extremal index is used for the Poisson-GP model and the value of this index should be close to 1 for a suitable threshold. More details on the selection of the threshold can be found in (Coles 2001).

The parameters of the distributions were estimated using the maximum likelihood estimation technique (MLE). The MLE approach is one of the most common used parameter estimation methods since it is straightforward to implement, unbiased and fully efficient (under regularity conditions) and normally distributed (in asymptotical sense).

The evaluation of the fitting of the various distributions was based on diagnostic plots (probability (p-p), quantile (q-q) and density) and the Kolmogorov-Smirnov goodness of fit test. Concerning the POT approach apart from the above-mentioned, evaluation metrics such as Akaike Information Criterion (Akaike 1974) were used for the selection of the best model. According to these criteria the most suitable statistical model would be the one with the lowest values.

3 Results

The samples extracted for each station in the annual maxima approach were checked for randomness, change points in the mean and monotonic trends. The results showed that the assumptions are valid for the samples of all stations according to the p-values of the tests. The p-values for all the tests are greater than 0.05 which corresponds to significance level 95%. Therefore, we can state that the null hypotheses of the tests are valid.

In the next step, according to the diagnostic plots and the Kolmogorov-Smirnov tests, all the distributions that were checked had an acceptable fitting in the sample of the three stations. This is clear in the results in Table 1 where the results of the Kolmogorov-Smirnov test are summarized. The distributions with the best fitting in each station are indicated in bolt letters, more specifically these are GEV for Larissa and Trikala and 3-par-Lognormal for Aghialos. Moreover, 3-parameter distributions perform a better fitting to the samples than the 2-parameter distributions since the D statistic of the test is lower for the 3-parameter distributions. This is also clear in diagnostic plots which are not presented here due to limited space.

Table 1 Kolmogorov Smirnov results for the various fitted distributions for the three stations.

Distributions		D		Dcrit
	Larisa	Aghialos	Trikala	
GEV	**0.07**	0.06	**0.09**	0.174
Gumbel	0.13	0.09	0.12	0.174
Exp	0.11	0.08	0.14	0.174
Gamma	0.13	0.10	0.15	0.174
Lognormal	0.11	0.08	0.12	0.174
Lognormal_3_par	0.09	**0.05**	0.11	0.174
Logistic	0.13	0.11	0.12	0.174
Loglogistic	0.09	0.07	0.09	0.174
PearsonIII	0.12	0.07	0.12	0.174

In Figure 1 the estimated rainfall depths for various return periods (10, 20, 30, 50, 100, 250 and 500 years) in each station are presented. The different colored curves correspond to the different distributions that were used. The various distributions estimate similar rainfall depths for small return periods (<30 years). However, as the return period increases the differences in the rainfall depths estimations among the distributions increase. This pattern is clear in all the stations. Statistically, this is explained by the fact that the various distributions are characterized by a different number of parameters. In fact, the values of the estimated parameters differ and consequently the estimated rainfall depths widely vary. In addition, the different shape of the distributions explains the differences in the estimated rainfall depths among the nine distributions. For example, upper tail distributions like GEV estimate larger rainfall depths as the return period increases. Moreover, this is further supported by the fact that some of the models were better fitted to the sample than others. A good fitting is present in the center of the sample for most of the distributions, but, only few of them were able to have a good fitting in the most extreme values of the sample which correspond to high return periods (>100 years). Moreover, the 2-parameter distributions underestimate the rainfall depths for high return periods comparing to the 3-parameter ones.

In Table 2 the estimated rainfall depths for the distributions presenting the best fitting in each station are given. The results are quite similar for the stations of Larissa and Aghialos except for very high return periods (>250 years) and they vary from 75 mm (10 years) to 230~260 mm (500 years). This is rational, since larger rainfall depths correspond to lower probabilities of recurrence (T=1/P, P=rank/n+1). Therefore, rainfall depths are increasing as the return period increases. Concerning the station in Trikala the estimated rainfall depths are larger than the other two for small return periods (<30 years) and smaller for high return periods (>100years) comparing to the other two stations.

Concerning the POT approach, precipitation values that correspond to the 99% percentile were selected as suitable thresholds based on the plots (MRL and parameters plots) and the evaluation metrics (AIC). No results are shown here due to limited space. In Figure 2 the estimated rainfall depths for the two models considered in the POT approach for the three stations are shown. A curve corresponding to the probability distribution with the best fitting in the first method is also plotted in order to make comparisons.

Table 2 Estimated rainfall depths for the distributions with the best fitting in each station.

Return Periods/Distributions	10	20	30	50	100	250	500
GEV (Larissa)	74.3	94.4	108.1	127.7	159.4	212.6	263.5
3-par-Lognormal (Agxialos)	76.4	96.7	110.2	127.8	154.9	195.9	231.3
GEV (Trikala)	87.0	102.7	112.6	125.6	144.9	173.7	198.2

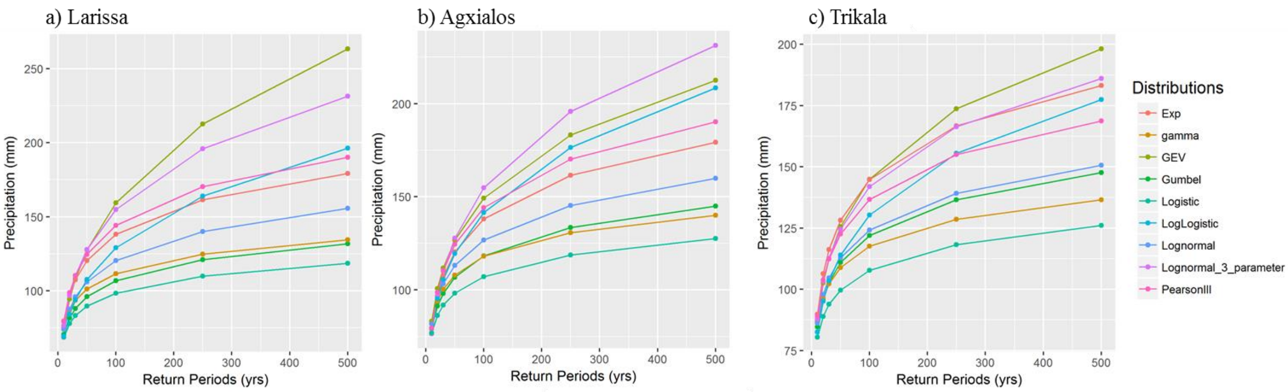

Fig. 1. Estimated rainfall depths for the various return periods for the three stations under investigation using annual maxima approach a) Larissa, b) Aghialos and c) Trikala.

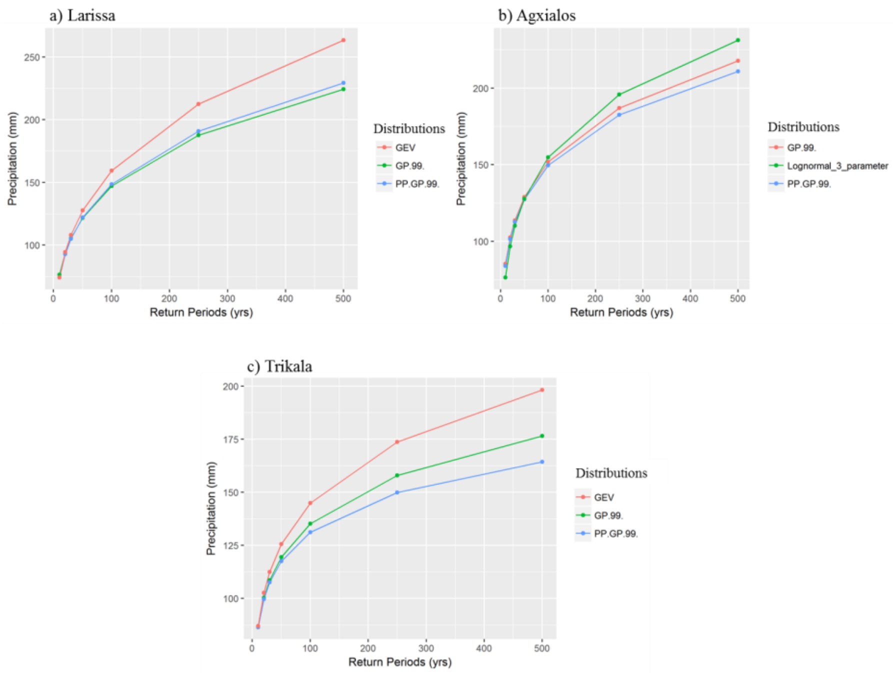

Fig. 2. Estimated rainfall depths for the various return periods for the three stations under investigation using peaks over threshold approach. a) Larissa, b) Aghialos and c) Trikala.

The results show that the two models, namely, GP and the mixed Poisson-GP model give almost the same results for all return periods except for the case of Trikala station where there are differences mainly for high return periods (>100 years). The two models also have similar results with the case of annual maxima especially for Aghialos station. However, differences between the two methods can be observed for high return periods (>100 years) in the stations of Larissa and Trikala.

4 Conclusions

In this study, statistical frequency analysis of extreme rainfall was performed using BM and POT approaches. The analysis was based on daily rainfall data from three stations in Thessaly region in Greece.

The various distributions that were fitted in the BM approach show similarities in the estimated rainfall depths for small return periods (<50 years). The differences among the distributions increase as the return period increases especially for high return periods (>100 years). The 3-parameters distributions that were studied had a better fitting in the samples of the three stations than the 2-parameter ones. GEV was the distribution with the best fitting in Larissa and Trikala Stations and the three-parameter lognormal distribution for Aghialos Station.

The two models examined in the POT approach give almost the same results for all the return periods except for Trikala Station where differences can be observed for high return periods.

The two approaches give similar results in the three stations especially for Aghialos. However, there are differences for high return periods in Larissa and Trikala Stations.

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