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# Comparison of various Sunk Cost Methods of Transmission Pricing

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**Abstract**—Issue of transmission pricing is a largely debated issue across the deregulated power systems. This is because, any transmission pricing scheme has to obey certain principles. The principles are contrasting and no single method can satisfy these principles. Every system tries to accommodate those principles that are based on prevailing conditions. Hence, there is no method that can be termed as ‘the best’ method. One has to make judicious choice between the methods. This paper compares a few popular methods for allocating transmission network cost to the constituents based upon various algorithms that find out ‘extent of use’ of the constituents. The results are compared for the locational variation under constrained and unconstrained cases. Thus, an attempt has been made to capture the congestion signal in transmission pricing method. The aim here is to determine that method/methods whose results are a tradeoff between the results from, the method that provides the ideal locational signal (pLMP) and the one that is commonly used to simplify the settlement process (Postage Stamp).

Based on the findings, certain important conclusions are drawn for suitability of a particular method/s. The results have been obtained on IEEE 30 bus system.

**Keywords**—Transmission Pricing, Postage Stamp, Real Power Tracing, Marginal Participation Factors (MAPF), Hybrid, Distributed Slack Bus MAPF, Locational Marginal Pricing, Sunk Cost

## NOMENCLATURE

$\beta$	Ratio of cost recovered from generators to the total cost
$GD_{ij}$	Equivalent exchange between ‘ $i^{th}$ ’ generator and ‘ $j^{th}$ ’ load
$LMP_{i,s}$	Locational Marginal Price at ‘ $i^{th}$ ’ bus for scenario ‘s’
$ND_i^{pLMP}$	Normalized difference with respect to pLMP at ‘ $i^{th}$ ’ bus
$ND_i^{PS}$	Normalized difference with respect to PS at ‘ $i^{th}$ ’ bus
$P_d^j$	Demand in MW at ‘ $j^{th}$ ’ bus

$P_d^{sys}$	Total system load
$P_g^i$	Generation in MW at ‘ $i^{th}$ ’ bus
$PI_1^c$	Performance index 1 for constrained network conditions
$PI_2^c$	Performance index 2 for constrained network conditions
$PI_1^u$	Performance index 1 for unconstrained network conditions
$PI_2^u$	Performance index 2 for unconstrained network conditions
$P_{Li}$	Load in MW at ‘ $i^{th}$ ’ bus
$pLMP_{i,s}$	Pseudo Locational Marginal Price at ‘ $i^{th}$ ’ bus for scenario ‘s’
$R_i$	Rate in Rs./MW at ‘ $i^{th}$ ’ bus
$R_i^{pLMP}$	pLMP rate in Rs./MW at ‘ $i^{th}$ ’ bus
$R_i^{PS}$	Postage stamp rate in Rs./MW at ‘ $i^{th}$ ’ bus
Slack $G_i$	Proportion of ‘ $i^{th}$ ’ generator in total generation
$TC_L$	Transmission cost to be paid by loads
$TC_{lm}$	Cost of transmission line ‘lm’

## I. INTRODUCTION

TRANSMISSION cost allocation is a highly debated and discussed topic in deregulated power industry. Vast amount of literature available in this area augments this fact. After the introduction of deregulation and unbundling of vertically integrated utilities transmission business has seen a drastic change in its way of operation. Suddenly the need of cost recovery and revenue generation has come into the limelight which was earlier a lookout of the vertically integrated utility in whole. Transmission charges constitute a small percentage of total operating expenses of a power utility. Nevertheless, a strong transmission network forms the backbone of competitive electricity markets. In a restructured power system *the transmission network is where generators compete to supply large users and distribution companies*. Thus transmission

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pricing should be a reasonable economic indicator used by the restructured power market to make decisions on resource allocation, system expansion and reinforcement [1], [2].

According to general transmission pricing guidelines laid down in [3], a transmission pricing scheme should,

- 1) Promote the efficient day-to-day operation of the bulk power market
- 2) Signal locational advantages for investment in generation and demand
- 3) Signal the need for investment in the transmission system
- 4) Compensate the owners of existing transmission assets
- 5) Be simple and transparent
- 6) Be politically implementable

All the transmission pricing methods can be broadly classified into two classes, **point-to-point** and **point-of-connection**. The point-to-point tariff is also known as **transaction based** tariff, covers methods like MW-mile (and all its variants like MVA-mile), postage stamp, contract path method (and its variants). These are essentially **embedded cost** methods. They do not offer any encouragement for efficient operation, nor give locational price signals as they do not consider any network usage. Various embedded cost methods for sunk cost recovery are discussed in details in [4]–[7]

The point-of-connection tariff is a relatively new concept. There is not much literature available on point-of-connection tariff philosophy [8]. This philosophy is used in Nordic power pool. The idea of the point-of-connection tariff is that “*the producers are paying a single charge to the grid for each kWh that they pour into the grid and the end users pay single charge for each kWh that they draw off the grid*” [9]. This charge is decided by the connection level of that particular entity. The distinguishing feature of the philosophy is that it can be applied for power exchange as well as for bilateral trades, which is a desired feature in the Indian context. This is essentially a **non-transaction based** tariff.

Many utilities are gradually shifting from traditional transaction based method to usage based methods. It is impossible to design a transmission pricing scheme which will follow all the guidelines mentioned above. In such a case it is essential to analyze and compare various methods and find a method which will strike the best compromise to incorporate most of the desired features. A flexible mix and match approach is used for development of a new transmission pricing scheme by giving suitable weights to all such desired features in [10]. This paper compares five different methods on the basis of the performance indices defined in section III-C

The paper is organized as follows: Section II gives a brief discussion about the transmission pricing methods being analyzed and compared. Section III constitutes of the system description, results and comparison of performance of the methods. The discussion of the results and important inferences are drawn in Section IV and Section V concludes the paper.

## II. TRANSMISSION PRICING METHODS

### A. Postage Stamp (PS) Method

This is the oldest, simplest and probably the crudest method of all. It does not require any power flow calculations and cost is allocated to each node based on the proportion of the nodal power (generation or load) to the total power. This method is politically popular due to its simplicity but it does not take into account the actual usage of the system. The system usage is considered on the averaged basis [1], [2].

### B. Power Tracing or Average Participation Factors (APF) Method

This class of transmission pricing algorithms is based on the so called Proportional Sharing Principle (PSP), which states that, “*the nodal inflows are shared proportionally among nodal outflows*”. As the name suggests, these algorithms actually trace the flow of power from generators to loads (upstream looking algorithm) and vice versa (downstream looking algorithms). This requires either AC or DC power flow to be done before implementing the algorithm. These algorithms provide us with four quantities, contribution of generators in line flows, distribution of loads in line flows, load-generation interaction and loss allocation.

There are two types of APF algorithms, simultaneous equations based and graph theory based. Bialek’s algorithm [11], [12] is an example of simultaneous equations approach where as Kirschen’s [13] and Wu’s [14] algorithms are of the latter type. Wu’s algorithm is used in this paper. The application of this algorithm to Western Regional Grid of India is given in [15]

### C. MArginal Participation Factors (MAPF) Method

This is a sensitivity factors based method, uses “*extent of use*” criterion for cost allocation. The usage is defined as incremental, i.e., the incremental power flow change in each corridor (or line) is computed for a 1 MW incremental change of demand or generation at each node. Once the power flow variation in each corridor incurred by each agent and for every scenario is obtained, it is possible to compute a yearly usage index for each network user as given in [16], [17]. Generally only positive changes in line flows are taken for calculation, negative ones are neglected, and this is how it has been used worldwide. But, it is possible to develop an algorithm considering negative flows and by crediting them instead of charging.

This method is used in *Chile* and *Argentina* where it is also known as the “*areas of influence*” method. This method is highly debatable due to its slack bus dependency. Results vary every time the slack bus is changed in power flow calculations.

### D. Hybrid Method

This method is used in India for calculation of nodal charges. This is a combination of APF method and MAPF method. The slack bus dependency of MAPF method is removed to a large extent by creating a distributed slack

bus. The load generation interaction results obtained from tracing algorithm are used to find out which generators are supplying which loads. These generators are designated as slack generators for incremental change in that particular load. Rest of the process is same as that of MAPF Algorithm. The creation of distributed slack bus removes the slack bus dependency of MAPF method to a large extent.

The development of this algorithm and its adoption in Indian context are discussed in detail in [18], [19].

#### E. Equivalent Bilateral Exchange (EBE) Method

This method developed by A. Conejo et al is based on principle of equivalent bilateral exchanges (EBE), "since a solved optimal power flow meets the laws of Kirchhoff without violating any line flow or generation limit, each generation injection flows without impediment toward all of the demands, while each demand is fed by all injected generations. As such, each demand is supplied by a fraction of each generator uniformly divided among all generators. Analogously, each generator supplies a fraction of each demand uniformly divided among all demands" [20]. The equivalent bilateral exchange between a generator at bus 'i' and a load at bus 'j' is given by (1),

$$GD_{ij} = \frac{P_{G_i} P_{D_j}}{P^{sys}_D} \quad (1)$$

This provides a simple way of calculating load generation interaction. The usage is calculated using the concept of Generalized Generation Shift Distribution Factors (GGSDF- $\gamma_{ij,k}$ ) [21].

#### F. Distributed Slack Bus MAPF (DMAPF)

This approach is also a variant of MAPF method discussed in II-C. In this method, a distributed slack bus is created using the proportion of individual generators in total generation. The generators are designated in this proportion to be the slack bus for any change in load.

If there are ' $n_g$ ' generators in a system the slack bus proportion is given by (2),

$$\text{Slack } G_i = \frac{G_i}{\sum_{i=1}^{n_g} G_i} \quad (2)$$

#### G. Pseudo Locational Marginal Price (pLMP)

The concept of **Pseudo-LMP** is introduced and developed in [8]. The  $LMP_{i,s}$  at a particular bus 'i' is the locational marginal price or **Short Run Marginal Cost (SRMC)** of that bus for scenario 's'. Being composed of cost of congestion, LMP gives the best reflection of cost of congestion at a particular node for scenario 's'. Hence this is the criterion against which the performance of all the methods discussed above will be checked. The objective is such that the spatial variation provided by any method must match as closely as possible with the spatial variation provided by  $LMP_{i,s}$ .

Let, TC be the total transmission cost, then the transmission cost to be paid by loads is given by,

$$TC_L = \sum_{\forall lm} \beta TC_{lm} \quad (3)$$

Where,

$TC_{lm}$  is the transmission cost of corridor 'lm',  $\beta$  is the fraction of the total cost to be paid by loads and hence  $(1-\beta)$  is the fraction of the total cost to be paid by generators. In our case  $\beta$  is taken as 0.5.

Let,  $pLMP_{i,s}$  represent the pseudo-LMP at node 'i' for scenario s, such that,

$$\sum_{i=1}^n pLMP_{i,s} \cdot P_{Li} = TC_L \quad (4)$$

The pLMP can be viewed as multiplying LMP by a factor ' $\alpha$ ' such that,

$$\sum_{i=1}^n \alpha \cdot LMP_{i,s} \cdot P_{Li} = TC_L \quad (5)$$

Hence,

$$pLMP_{i,s} = \alpha \cdot LMP_{i,s} \quad (6)$$

This pseudo-LMP has the same spatial variation as that of LMP for a particular scenario. LMP is an *ex-post* price indicator and hence the prices are not known to users *a-priori*, but, the transmission pricing methods analyzed are *ex-ante* schemes which make prices known to users beforehand. Hence a transmission pricing method as close as possible to LMP spatial distribution pattern would be the best method as it would generate required price signals beforehand.

Also, this scheme does not accommodate the usage of the network by participants, as '*extent of use*' of the network by participants is not quantified. Calculation of '*extent of usage*' is important because it relates the sunk costs of the transmission network to the point charges.

In ideal condition the locational price signals provided by a method should have zero difference with the pLMP values per node. The objective is to find the method having least difference and which allocates the sunk cost to the participants based on the usage of the network by them.

### III. CASE STUDY

#### A. System Description

Fig. 1 shows the standard IEEE 30 bus system used for implementation and study of all the transmission pricing methods discussed so far.

The data for this system is available in [22].

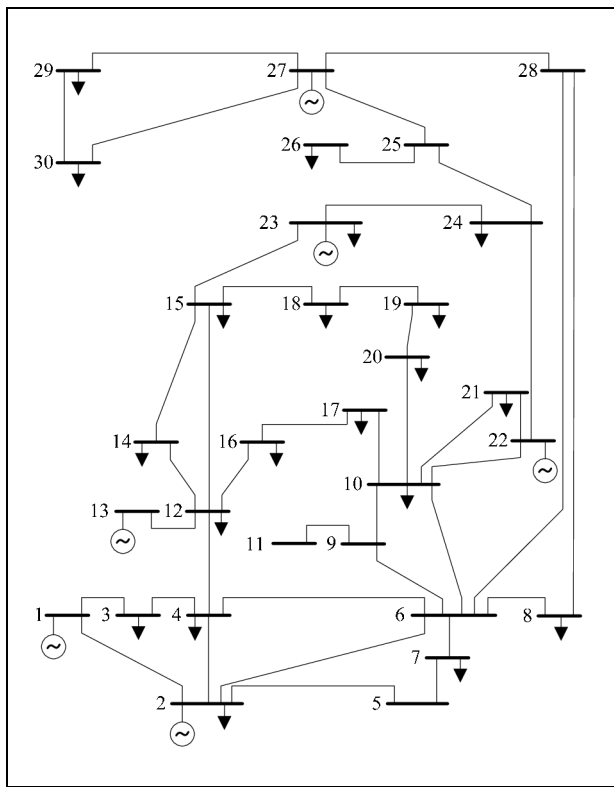


Fig. 1: Standard IEEE 30 bus system

The common basis used for analysis is as follows,

- System is assumed to be lossless (R neglected)
- DC Optimal Power Flow (DCOPF) is used as the power flow method
- The sunk cost of the whole transmission network is assumed to be Rs. 10,00,000
- Half of the cost is to be recovered from generators and half from the loads ( $\beta=0.5$ )

Two scenarios are generated:

Scenario 1: An unconstrained case, where transmission lines are assumed to have sufficient power carrying capacity,

Scenario 2: Constrained case, where constraints on some line flows are deliberately put in order to create a congested power flow case.

### B. Results

Figure 2 provides nodal charges for all the methods including pLMP and Postage Stamp method, for Scenario 1.

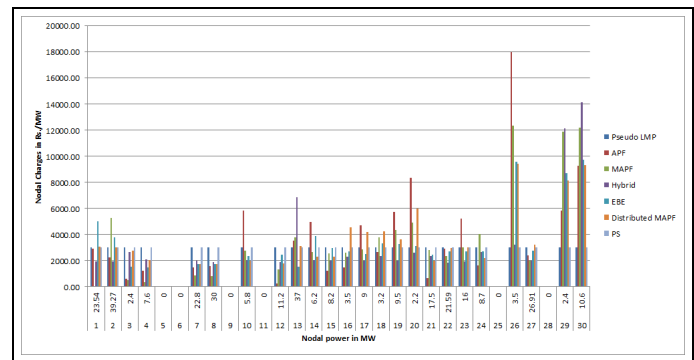


Fig. 2: Graphical Illustration of Nodal Charges in Rs./MW Obtained from Different Methods under Unconstrained Network Conditions

Figure 3 provides nodal charges for all the methods including pLMP and Postage Stamp method, for Scenario 2.

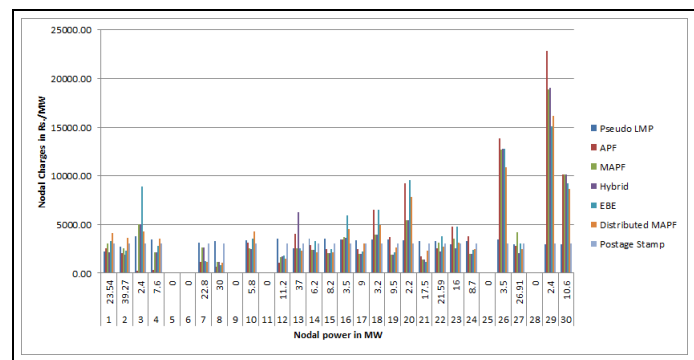


Fig. 3: Graphical Illustration of Nodal Charges in Rs./MW Obtained from Different Methods under Constrained Network Conditions

The loads at nodes 26, 29 and 30 are low on magnitude but are being served by very long lines (which is evident from reactance values of these lines which are highest among all) with high sunk cost. This is very well reflected in the high rates at these nodes, which is depicted in the Fig.2 and 3

### C. Comparison

To check the performance of the methods, the methods are compared with respect to:

- 1) Extent of locational variation in prices,
- 2) Closeness of the results to optimal price signals, and
- 3) Closeness of the results with respect to flat rate.

TABLE I: Comparison of Nodal Rates in Rs./MW under Unconstrained Network Conditions

Parameter	PS	APF	MAPF	Hybrid	EBE	DMAFP	pLMP
Average rate	3043.12	3952.36	3748.87	3290.09	3534.17	3708.19	3043.12
Standard Deviation ( $\sigma$ )	0	3849.92	3526.60	3212.94	2395.84	2283.47	0

TABLE II: Comparison of Nodal Rates in Rs./MW under Constrained Network Conditions

Parameter	PS	APF	MAPF	Hybrid	EBE	DMAFP	pLMP
Average rate	3043.12	4851.74	4395.99	4330.83	4981.83	4539.33	3237.41
Standard Deviation ( $\sigma$ )	0	5091.17	4118.04	4238.11	3816.17	3479.42	343.75

Table I & II show a reversal in trend of rates for EBE and DMAFP method. This reversal is due to the congestion condition introduced in the network. The EBE method allocates all the generators to all the loads in the same proportion and does not take into account the sensitivity of different loads and generators for change in transmission line power carrying capacity gives lower average rates during unconstrained conditions. The DMAFP method, being based on sensitivity factors and distributed slack bus principle allocates the generators to all loads in same proportion as that of EBE but it also takes into account sensitivity of loads and generators for change in transmission line power carrying capacity. This gives lower average rates for DMAFP method under unconstrained network conditions. Hence there is a reversal of average rates under two different network conditions.

Performance indices  $PI_1$  and  $PI_2$  are defined to compare the performance of all these methods against the pLMP and postage stamp rates for both the network conditions. These performance indices are the mean and standard deviation of the normalized difference respectively. The normalized difference is given by (7) and (8),

$$ND_i^{pLMP} = \frac{|R_i^{pLMP} - R_i|}{R_i} \quad (7)$$

$$ND_i^{ps} = \frac{|R_i^{ps} - R_i|}{R_i} \quad (8)$$

Where,  $ND_i$  is the normalized difference for  $i^{th}$  node.  $R_i^{ps}$  and  $R_i^{pLMP}$  are nodal rates in Rs./MW for  $i^{th}$  node by postage stamp method and pLMP respectively.  $R_i$  is the nodal rate obtained by a method in Rs./MW for  $i^{th}$  node.

The performance indices are given by (9) and (10)

$$PI_1 = \frac{\sum_{i=1}^n ND_i}{n} \quad (9)$$

$$PI_2 = \sigma(ND_i)_{i=1}^n \quad (10)$$

where,

$PI_1^u$  and  $PI_2^u$  are the performance indices 1 and 2 for a method under unconstrained network conditions, similarly  $PI_1^c$   $PI_2^c$  are the performance indices for constrained network conditions.

'n' is the number of nodes

A comparison is made for the average and standard deviation of nodal charges obtained by all the methods for unconstrained (Table:I) and constrained network condition (Table:II)

TABLE III: Comparison of Performance Indices  $PI_1^u$  and  $PI_2^u$  for Unconstrained Network Conditions

Parameter	APF	MAPF	Hybrid	EBE	DMAFP
$PI_1^u$ (Mean)	1.33	1.04	0.46	0.38	0.35
$PI_2^u$ ( $\sigma$ )	2.45	1.81	0.19	0.34	0.26

Since pLMP and Postage stamp rates are same for unconstrained network conditions only one comparison is made.

Comparison of performance indices with pLMP for constrained network conditions is given in table IV.

TABLE IV: Comparison of  $PI_1^c$  and  $PI_2^c$  for all Methods with respect to pLMP

Parameter	APF	MAPF	Hybrid	EBE	DMAFP
$PI_1^c$ (mean)	1.693	0.53	0.59	0.62	0.52
$PI_2^c$ ( $\sigma$ )	3.485	0.45	0.42	0.68	0.53

Comparison of performance indices with PS rates is given in table V.

TABLE V: Comparison of  $PI_1^c$  and  $PI_2^c$  for all Methods with respect to PS

Parameter	APF	MAPF	Hybrid	EBE	DMAFP
$PI_1^c$ (mean)	1.463	0.47	0.54	0.59	0.47
$PI_2^c$ ( $\sigma$ )	2.875	0.38	0.35	0.61	0.47

#### IV. DISCUSSION

The sunk cost allocation methods discussed can be employed for ex-ante or ex-post pricing of transmission network. Thus, any of these methods can be employed to establish Point-of-Connection (PoC) rates. The simulations of sunk cost methods - APF, MAPF, Hybrid, EBE, DMAFP on IEEE 30 bus system are carried out and the results are compared on account of the following attributes:

The methods are compared with respect to:

- 1) Extent of locational variation in prices,
- 2) Closeness of the results to optimal price signals,
- 3) Closeness of the results with respect to flat rate.

When it comes to gauging vagaries in locational pricing, the DMAFP method provides best results. This becomes apparent from the standard deviation figures in tables I and II. The issue of large variation of prices across the system is a very sensitive

issue, especially in the systems where epoch change is being made in transmission pricing. These systems earlier employed postage stamp method and a sudden locational variation in prices creates a set of winners and losers. Hence, to have all constituents on board, the policy makers of such systems wish to employ that method which would provide moderate signals without large variations in locational prices. It can be seen that the DMAPF and EBE methods satisfy this criteria to a good extent.

The closeness of transmission pricing rates to optimal price signals is another important issue of transmission pricing methods. The optimal short run price signals are depicted by pLMP. From table IV, the methods can be ranked (with respect to above mentioned criteria) as follows:

- 1) Hybrid
- 2) MAPF
- 3) DMAPF

A version of Hybrid method discussed in this work is currently employed in India. However, it involves calculation of two methods - Tracing and Marginal participation. In order to avoid criticism against the slack dependent results, the power flow tracing is carried out first to select slack buses. This provides a scientific rationale for choice of slack bus. However, the whole proposal becomes complex, as two stand-alone methods need to be simulated. This is against the principle of simplicity and transparency. Thus, the hybrid method, though works out well for optimality of the rates, fares badly when it comes to another principle of transmission pricing - simplicity. The MAPF, though ranked second in this regard is cursed by choice of arbitrary slack bus and same results can not be guaranteed for some other choice of slack bus. Thus, arbitrariness in choosing slack bus is lacuna associated with this method. DMAPF, on the other hand provides an intuitive way of choosing slack and thus becomes easy to implement compared to Hybrid method. Thus, DMAPF works well when principle of simplicity is concerned.

The reason behind evaluating closeness to postage stamp rates is again useful in systems that are undergoing changes from postage stamp rates to some usage based method. The lesser the deviation from that of postage stamp rates, more is the acceptance of such method amongst stake-holders. From table III, it can be seen that under unconstrained case, Hybrid and DMPAF methods provide good results on the above aspect. However, under congested case, Hybrid, MAPF and DMAPF provide close results to postage stamp rates (Table V). Depending on the congestion history of the system, the policy maker can adopt appropriate method.

The above discussion and the associated results lead us to the fact that the DMAPF method is a least common multiple amongst all the methods. It shows a better promise so long as satisfying some of the important principles of transmission pricing are concerned.

The centralized despatch systems calculate LMPs which are the most efficient price signals. The de-centralized despatch systems do not calculate LMPs and the efficient price signals are unavailable. Here, an attempt has been made to suggest a pricing method that would show results close to most efficient

signals. In the absence of LMP calculation mechanism, the said method could be employed.

The systems that used postage stamp method and wish to change their transmission pricing regime, they want to place a scheme that provides mild locational signals, so that there are no financial shocks to stake-holders. The DMAPF method, by virtue of its closeness to postage stamp method establishes itself as good candidate to be employed in such systems.

## V. CONCLUSION

This paper provides a comparison of sunk cost allocation methods from a utility's point of view which wishes to upgrade to a usage based transmission pricing methodology from a simple and politically acceptable Postage Stamp method. Such a scenario would require a method which will give mild locational signals as well as will reflect the system usage. It is observed from the results that the DMAPF method makes the best compromise between these requirements along with the additional advantage of relative simplicity as compared to other methods. This compromise is vital, in order to create minimum shocks for the stake holders .

This study is also important from practical point of view since many utilities in India still use the old postage stamp method and will be upgrading for a suitable usage based method in near future. All the methods discussed so far focus solely on the real power transactions and trade as the sunk cost and system usage are directly linked to the real power flow on the network. Also, pricing of reactive power and its trade is beyond the scope of this work.

The work has been carried out on the standard IEEE 30 bus system in the hope that it will prove as a pilot study for future work. A vast territory still remains unexplored in the form of the implementation and comparison of these methods on real life systems.

## REFERENCES

- [1] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems Forecasting, Scheduling, and Risk Management*. John Wiley & Sons Ltd, 2002, ISBN: 0-471-22412-X.
- [2] M. Shahidehpour and M. A. Yarmouk, *Restructured Electrical Power Systems Operation, Trading, and Volatility*. Power Engineering, Marcel Dekker, Inc., 2004, ISBN: 0-8247-0620-X.
- [3] R. Green, "Electricity transmission pricing: an international comparison," *Utilities Policy*, vol. 6, no. 3, pp. 177 – 184, 1997.
- [4] J. Lima, "Allocation of transmission fixed charges: an overview," *IEEE Transactions on Power Systems*, vol. 11, no. 3, pp. 1409–1418, 1996.
- [5] D. A. Lima, A. Padilha-Feltrin, and J. Contreras, "An overview on network cost allocation methods," *Electric Power Systems Research*, vol. 79, no. 5, pp. 750 – 758, 2009.
- [6] Z. Jing, X. Duan, F. Wen, Y. Ni, and F. Wu, "Review of transmission fixed costs allocation methods," in *Power Engineering Society General Meeting, 2003, IEEE*, vol. 4, pp. –2592 Vol. 4, 2003.
- [7] D. Shirmohammadi, X. Filho, B. Gorenstin, and M. Pereira, "Some fundamental, technical concepts about cost based transmission pricing," *IEEE Transactions on Power Systems*, vol. 11, no. 2, pp. 1002–1008, 1996.
- [8] A. Abhyankar and S. Khaparde, "Electricity transmission pricing: Tracing based point-of-connection tariff," *International Journal of Electrical Power & Energy Systems*, vol. 31, no. 1, pp. 59 – 66, 2009.

- [9] "Nord pool spot, how does it work - point tariff system."
- [10] V. N. Bharatwaj, A. Abhyankar, and P. Bijwe, "Flexible mix and match approach for network fixed cost allocation," *International Journal of Electrical Power & Energy Systems*, vol. 42, no. 1, pp. 8 – 15, 2012.
- [11] J. Bialek, "Tracing the flow of electricity," *IEE Proceedings-Generation, Transmission and Distribution*, vol. 143, no. 4, pp. 313–320, 1996.
- [12] J. Bialek, "Topological generation and load distribution factors for supplement charge allocation in transmission open access," *IEEE Transactions on Power Systems*, vol. 12, no. 3, pp. 1185–1193, 1997.
- [13] D. Kirschen, R. Allan, and G. Strbac, "Contributions of individual generators to loads and flows," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 52–60, 1997.
- [14] F. Wu, Y. Ni, and P. Wei, "Power transfer allocation for open access using graph theory-fundamentals and applications in systems without loopflow," *IEEE Transactions on Power Systems*, vol. 15, no. 3, pp. 923–929, 2000.
- [15] A. Roy, A. R. Abhyankar, P. Pentayya, and S. Khaparde, "Electricity transmission pricing: tracing based point-of-connection tariff for indian power system," in *Power Engineering Society General Meeting, 2006. IEEE*, pp. 8 pp.–, 2006.
- [16] F. Rubio-Oderiz and I. Perez-Arriaga, "Marginal pricing of transmission services: a comparative analysis of network cost allocation methods," *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 448–454, 2000.
- [17] I. Perez-Arriaga, F. J. Rubio, J. F. Puerta, J. Arceluz, and J. Marin, "Marginal pricing of transmission services: an analysis of cost recovery," *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 546–553, 1995.
- [18] C. E. R. Commission, "Formulating pricing methodology for inter-state transmission in india, attachment-i to the central electricity regulatory commission (sharing of transmission charges and losses) regulations," tech. rep., CERC, 2010.
- [19] C. E. R. Commission, "Simulation of hybrid method for determination of point of connection (poc) based transmission charges on 2011 & 12 indian power system," tech. rep., CERC, 2010.
- [20] F. Galiana, A. Conejo, and H. Gil, "Transmission network cost allocation based on equivalent bilateral exchanges," *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1425–1431, 2003.
- [21] W. Y. Ng, "Generalized generation distribution factors for power system security evaluations," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 3, pp. 1001–1005, 1981.
- [22] E. E. D. University of Washington, "Power System Test Case Archive." <http://www.ee.washington.edu/research/pstca/>.