# Some Applications of the Software GRAN1 to Analyze Two-Dimensional Continuous Probability Distributions

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# Some applications of the software GRAN1 to analyze two-dimensional continuous probability distributions

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**Abstract.** The article considers the use of GRAN1 software in the analysis of two-dimensional continuous probability distributions. The learning process can be provided by freely distributable software, through the use of cloud technologies, in particular software from the GRAN package. However, the use of any technology in the educational process, including modern information and communication technologies, as well as the content of education, must be pedagogically balanced, which will avoid any negative consequences for the formation of the future member of society, his mental and physical development. Attention is drawn to the fact that children are increasingly looking for entertainment, using mobile devices, sharing new products with their peers and spending time together, playing a variety of online games, and parents cannot control this process. At the same time, teachers conduct lessons using mobile devices to draw attention to the subject, they are teaching. In the process of using computer technology, students can form a special type of thinking, the so-called "clip consciousness", which leads to fragmentary thinking and the irreversibility of changes in consciousness. Therefore, the problem of excessive use of mobile devices by children requires constant analysis and coverage. The article discusses some examples of using the GRAN1 software to solve problems in probability theory. Examples of approximate calculation of the value of the double integral are given. Tables of values of the corresponding integrals are described. The use of modern information and communication technologies during mathematics lessons is considered.

## 1. Introduction

Significant spread of the use of modern information and communication, in particular cloud-oriented, technologies (see [6]) in the educational process, in particular in the process of teaching mathematics, requires the development of new computer-based teaching methods focused on the harmonious combination of pedagogical achievements past and present scientific and technical achievements, the pedagogically balanced use of various computer-based learning environments in the learning process, the formation of students primarily scientific worldview, analytical and synthetic, critical and logical, creative thinking, their mental, physical and general cultural development, correct vision of the world, careful attitude to the surrounding world and to people, benevolence, humanity, peace (see [6], [9]).

The purpose of writing this article is to introduce into the course of computational mathematics for students at pedagogical universities analysis of two-dimensional continuous probability distributions as a new fragment.

Of particular importance is the teaching of natural sciences and mathematics, during which students have to consider and build models of various processes and phenomena, and then explore them, analyzing their various features and characteristics, possibly using different information and communication models to perform calculations or experiments, and based on the results of such analysis,

synthesizing the relevant conclusions. This approach to learning allows students to effectively develop logical, critical, creative thinking, scientific worldview, creative approach to solving various problems, their correct vision and ability to explain their nature and essence.

Of particular importance are the cloud technologies for accessing the resources of various powerful computers (servers) via the Internet using not very powerful, including mobile Internet devices – smartphones, tablets, etc. This allows educational institutions to use the resources of remote servers without spending money to purchase their own powerful, and therefore expensive, computers. It is enough to have access to the Internet through the appropriate browser to get to the open virtual desktop on a powerful remote computer and then use the resources of the remote computer (server) to develop solving their problems in relation to the processing of various information resources – solving mathematical problems, processing texts, translation from one language to another, information on the interpretation of different terms, their origin, and much more [15].

On the example of using the resources of a remote server with "cloud" versions of the software package for educational purposes Gran solve several problems.

## 2. Theoretical background

Children are increasingly looking for entertainment using mobile devices, sharing new products with their peers and spending time together, playing a variety of online games. Be sure to consider the time spent on such devices to prevent getting used to the uncontrolled use of certain applications or due to exposure to unwanted content [1].

Teachers began conducting lessons using mobile devices and computers to draw attention to the subject. It should be understood, that the informatization of education is not a panacea, there are risks of loss of cultural and creative thinking, live communication, and the illusion of accessibility of cognitive actions. In the process of using computer technology, students form a special type of thinking, the so-called "clip consciousness", which leads to fragmentary thinking and the irreversibility of changes in consciousness. This causes a bad mood, irritability, often general malaise, or fatigue. As a result, appetite is reduced, sleep is disturbed, and efficiency is reduced.

The problem of overuse of mobile devices and computers requires constant analysis and coverage. It is clear, that the development, creation and use of software to control the time spent by children on gadgets is one of the urgent needs of parents and teachers to prevent the "collage of modern thinking" of children [2], [5].

However, with a harmonious combination, the educational process improves, because there is always a lively discussion between students. Researchers have shown that children of all ages memorize learning material best while playing. Therefore, motivated use of mobile devices and computers during learning increases student activity [3], [10].

One of the most effective ways to develop analytical, synthetic and logical thinking of students, as we know, is the teaching of mathematics, its various sections - geometry, algebra, mathematical analysis, probability theory, computational mathematics, discrete mathematics, real variable function theory and others [4], [11].

A special place among such sections is occupied by probability theory, its connections with the theory of measure, geometry, mathematical analysis, computational mathematics. Teaching the basics of probability theory to students at school and future teachers at the Pedagogical University is an important basis for the formation of their professional awareness and professional culture [8].

## 3. Results

Consider some examples of the use of software GRAN1 (see [4], [13], [15]) to solve problems in probability theory, in particular closely related to the corresponding problems in geometry and mathematical analysis [7], relating to the calculation of probabilities of hitting some two-dimensional sets for conditions of two-dimensional distribution of probabilities on the plane, which are reduced to the calculation of double integrals on convex sets, and hence to the calculation of the volumes of the corresponding spatial figures.

For an approximate calculation of the value of the double integral

$$\iint\limits_{G} f(x,y)\,dx\,dy,$$

where $f(x,y) \geq 0$ – integral function, $G$ – convex two-dimensional set, divide the set $G$ parallel equidistant lines of view $y = y_i$, $i \in \overline{1,k}$, on separate strips, and for each $y_i$ define $a(y_i)$ – the smallest value of the abscissa $x$ on the straight $y = y_i$ such that the point $(a(y_i), y_i)$ is the inner point of the set $G$, as well as $b(y_i)$ – the greatest value of the abscissa $x$ on the straight $y = y_i$ such that the point $(b(y_i), y_i)$ is the inner point of the set $G$, namely $a(y_i)$ – the leftmost point in the area $G$ on the straight $y = y_i$, $b(y_i)$ – the rightmost point in the area $G$ on the straight $y = y_i$. Value $y_i$, by which points $(a(y_i), y_i)$, $(b(y_i), y_i)$ are not internal points of the set $G$, are not included in the consideration. Next will present approximately (see [12])

$$\iint\limits_{G} f(x,y)\,dx\,dy$$

through

$$\sum_{i=1}^{k-1} h \int\limits_{a(y_i)}^{b(y_i)} (x,y_i)dx + h \cdot \frac{1}{2}\left( \int\limits_{a(y_0)}^{b(y_0)} f(x,y_0)dx + \int\limits_{a(y_k)}^{b(y_k)} f(x,y_k)dx \right) \qquad (1)$$

or what is the same, by the formula

$$\sum_{i=1}^{k} h \cdot \frac{1}{2}\left( \int\limits_{a(y_i)}^{b(y_i)} f(x,y_{i-1})\,dx + \int\limits_{a(y_i)}^{b(y_i)} f(x,y_i)\,dx \right) \qquad (2)$$

where $y = y_0$ – the lowest line, that touches the set $G$,

$y = y_k$ – the highest line that touches the set $G$,

$h = y_i - y_{i-1}$ – the width of the strip between the lines $y = y_i$ and $y = y_{i-1}$. Note that in the general case, the bands between the lines $y = y_i$ and $y = y_{i-1}$, $i \in \overline{1,k}$, may not be the same width. Then the appearance of the formula will change accordingly (1).

Let on the set $\Omega = \{(x,y) | x^2 + y^2 \leq 2^2\}$ the probability distribution is set due to the density of the probability distribution

$$f(x,y) = \begin{cases} \dfrac{3}{8\pi}\left(2 - \sqrt{x^2 + y^2}\right), & \text{when } (x,y) \in \Omega, \\ 0, & \text{when } (x,y) \bar{\in} \Omega. \end{cases}$$

In geometric interpretation through the specified function $f(x,y)$ describes a straight circular cone with a radius of the base $R = 2$ and height $h = \frac{6}{8\pi}$ (see Figure 1). It is easy to see the volume of such a cone $V = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi \cdot 4 \cdot \frac{6}{8\pi} = \frac{4\pi}{3} \cdot \frac{6}{8\pi} = 1$, namely in relation to the specified function $f(x,y)$ all properties of the density distribution are satisfied (see [14]):

1) $f(x,y) \geq 0$;

2) $\iint\limits_{R^2} f(x,y)dxdy = 1$.

As is known, in geometric interpretation in case of $f(x,y) \geq 0$

$$\iint\limits_{G} f(x,y)dxdy$$

is the volume of the body below the surface $z = f(x,y)$ over the area $G$.

Let for a given probability distribution with density

$$f(x,y) = \begin{cases} \dfrac{3}{8\pi}\left(2 - \sqrt{x^2 + y^2}\right), & \text{when } x^2 + y^2 \leq 2^2, \\ 0, & \text{when } x^2 + y^2 > 2^2, \end{cases}$$

need to calculate the probability of getting into the square

$$G = [-1; 1] \times [-1; 1] = \{(x,y)|x \in [-1; 1], y \in [-1; 1].$$
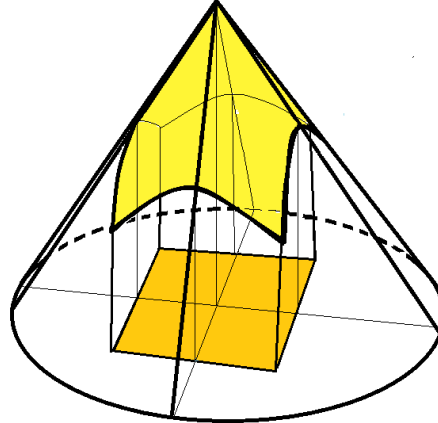


**Figure 1.** Intersection of two spatial figures.

In the geometric interpretation, this means that it is necessary to calculate the volume of the body, which is obtained as the intersection of two spatial figures – the specified straight circular cone and a straight parallelepiped with a base $G = [-1; 1] \times [-1; 1]$ and a height not less than the height of the cone. Note that in this case, the intersection of figures means the set of points that are the internal points of both one figure and another (reciprocally to the intersection of sets of points).

In other words, you need to calculate the volume of the body below the surface of the cone above the square $G = [-1; 1] \times [-1; 1]$, that is, inside the specified parallelepiped (see Figure 1).
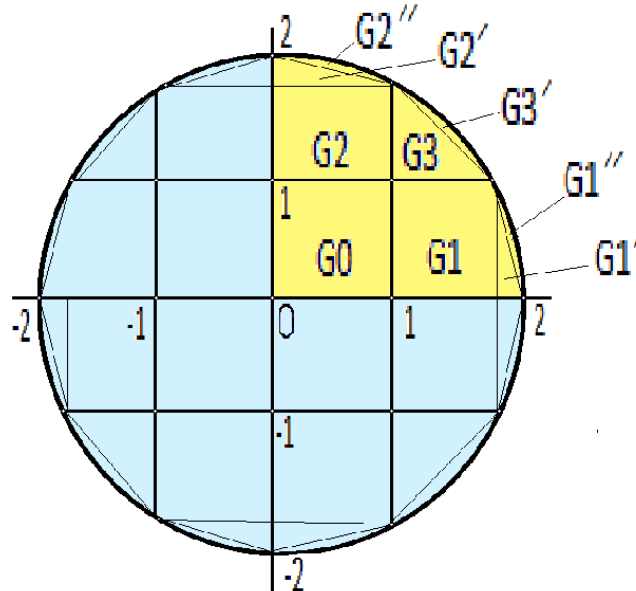


**Figure 2.** Probability of hitting the area $G$.

For an approximate calculation of the probability of hitting a given square $G = [-1; 1] \times [-1; 1]$ at a given density $f(x,y)$ probability distribution on the set $\Omega = \{(x,y)|x^2 + y^2 \leq 2^2\}$ first calculate the probability of hitting the area $G\emptyset = [0; 1] \times [0; 1]$ (see Figure 2) at the same density $f(x,y)$ probability distribution.

Given formula (1), we choose the step of changing the variable $y_i$ equal $h = 0.1$, and instead of a variable $y_i$ enter a dynamic parameter $p1$, which we will change from the value $p1 = 0$ through the step $h = 0.1$ to value $p1 = 1.0$ and for each parameter value so determined $p1$ we will calculate the integrals
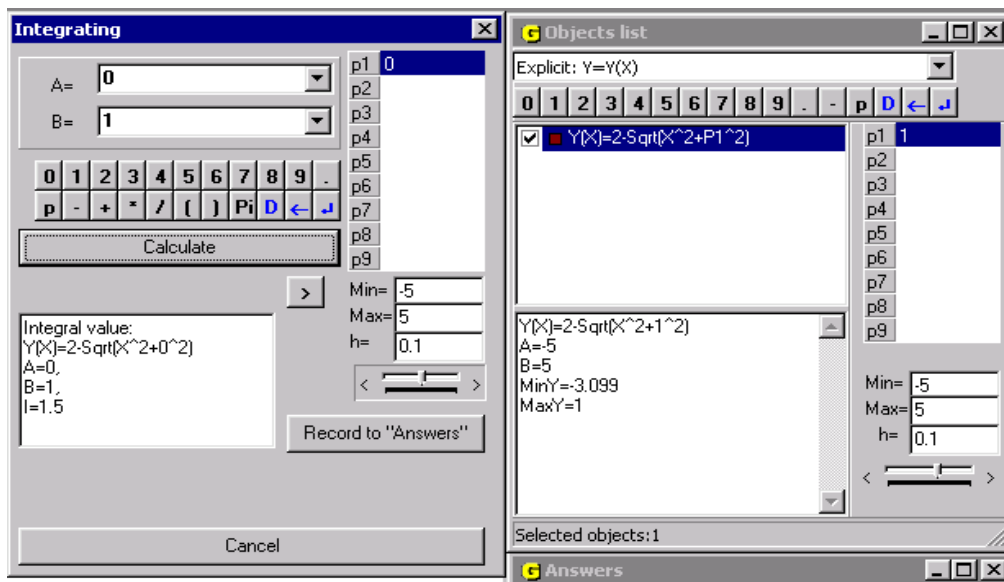


**Figure 3.** Calculation of integrals in the program.

$$\int_0^1 \left(2 - \sqrt{x^2 + p1^2}\right) dx,$$

using the relevant services of the program GRAN1 (see Figure 3).

As a result, we obtain a table of values of the corresponding integrals (see table 1)

**Table 1.** The value of the corresponding integrals.

| $p1$ | $\int_0^1 \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
|:---:|:---:|
| 0 | 1.5 |
| 0.1 | 1.483 |
| 0.2 | 1.444 |
| 0.3 | 1.392 |
| 0.4 | 1.33 |
| 0.5 | 1.261 |
| 0.6 | 1.186 |
| 0.7 | 1.107 |
| 0.8 | 1.024 |
| 0.9 | 0.9394 |
| 1.0 | 0,8522 |

Calculating the sum of the values of the integral thus found and subtracting from it the values multiplied by 0.5 values obtained at $p1 = 0$ and by $p1 = 1.0$ (see formula (1)), and in addition, multiplying the obtained result on the constant $\frac{3}{8\pi}$ and gave on 0.1, we will receive $P(G\emptyset) = 0.1473$.

Thus $P(G) = 4P(G\emptyset) = 0.5892$.

Note that for the calculation of the new value of the integral and for the new value of the parameter $p1$ there is no need to enter the expression of the sub integral function and the limits of integration each

time. After increasing the value of the parameter $p1$ on the step $h$ the limits of integration and the expression of the sub integral function change automatically and then the corresponding value of the integral is calculated.

Calculating $P(G1 \cup G1' \cup G1'')$, where
$$G1 \cup G1' \cup G1'' = \{(x, y) | x \in [1; 2], y \in [0; 1], x^2 + y^2 \leq 2^2\},$$
we obtain a table of values of the corresponding integrals (see table 2):

**Table 2.** The value of the corresponding integrals.

| $p1$ | $\displaystyle\int_{1}^{\sqrt{4-p1^2}} \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
|------|------|
| 0 | 0.5 |
| 0.1 | 0.4965 |
| 0.2 | 0.4863 |
| 0.3 | 0.4694 |
| 0.4 | 0.4465 |
| 0.5 | 0.418 |
| 0.6 | 0.3848 |
| 0.7 | 0.3475 |
| 0.8 | 0.3072 |
| 0.9 | 0.2648 |
| 1.0 | 0.2214 |

Calculating, as before, the sum of all values found and then subtracting half the sum of the two extreme values (see. formula (1)) and multiplying the amount thus obtained first by the constant $\frac{3}{8\pi}$, and then on $h = 0.1$, we will receive $P(G1 \cup G1' \cup G1'') = 0.04753$.

Quite similar in relation $P(G2 \cup G2' \cup G2'' \cup G3 \cup G3')$, where

$$G2 \cup G2' \cup G2'' \cup G3 \cup G3' = \{(x, y) | x \in [0; \sqrt{3}], y \in [1; 2], x^2 + y^2 \leq 2^2\},$$

we will receive table of values of the corresponding integrals (see table 3):

**Table 3.** The value of the corresponding integrals.

| $p1$ | $\displaystyle\int_{0}^{\sqrt{4-p1^2}} \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
|------|------|
| 1.0 | 1.074 |
| 1.1 | 0.9413 |
| 1.2 | 0.809 |
| 1.3 | 0.6782 |
| 1.4 | 0.5506 |
| 1.5 | 0.4281 |
| 1.6 | 0.3128 |
| 1.7 | 0.2073 |
| 1.8 | 0.115 |
| 1.9 | 0.04142 |
| 2.0 | 0 |

Calculating the sum of all values found and then subtracting half the sum of the two extreme values and multiplying the result first by $\frac{3}{8\pi}$ and then on 0.1, we will receive

$$P(G2 \cup G2' \cup G2'' \cup G3 \cup G3') = 0.05502.$$

Folding the obtained probability values $P(G\emptyset)$, $P(G1 \cup G1' \cup G1'')$, $P(G2 \cup G2' \cup G2'' \cup G3 \cup G3')$, we will receive

$$0.1473 + 0.04753 + 0.05502 = 0.2499,$$

that is, the probability of getting into the set $\{(x,y)|x \geq 0, y \geq 0, x^2 + y^2 \leq 2^2\}$ (the upper right quarter of the circle) is approximately equal to $0.2499 \approx 0.25$, which indicates a fairly high accuracy of the calculations.

Obviously, in the considered case

$$P(G2 \cup G2' \cup G2'') = P(G1 \cup G1' \cup G1'') = 0.04753,$$

whence

$$P(G3 \cup G3') = P(G2 \cup G2' \cup G2'' \cup G3 \cup G3') - P(G2 \cup G2' \cup G2'') =$$
$$= 0.05502 - 0.04753 = 0.00749.$$

Because $G3 = \{(x,y)|x \in [1; \sqrt{3}], y \in [1; \sqrt{3}], x + y \leq 1 + \sqrt{3}\}$, then calculating as before, $P(G3)$, we get (see table 4)

**Table 4.** The value of the corresponding integrals.

| $p1$ | $\displaystyle\int_{1}^{1+\sqrt{3}-p1} \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
|---|---|
| 1.0 | 0.2214 |
| 1.1 | 0.1775 |
| 1.2 | 0.1343 |
| 1.3 | 0.094 |
| 1.4 | 0.05862 |
| 1.5 | 0.03007 |
| 1.6 | 0.01019 |
| 1.7 | 0.0006251 |
| 1.732 | 0.0000 |

and further, given formula (1) or (2), we find

$$P(G3) = 0.007353, \text{ whereof}$$

$$P(G3') = P(G3 \cup G3') - P(G3) = 0.00749 - 0.007353 = 0.000137.$$

Concerning $P(G1)$, where $G1 = \{(x,y)|x \in [1; \sqrt{3}], y \in [0; 1]\}$, we receive (see table 5).

**Table 5.** The value of the corresponding integrals.

| $p1$ | $\displaystyle\int_{1}^{\sqrt{3}} \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
|---|---|
| 0 | 0.4641 |
| 0.1 | 0.4614 |
| 0.2 | 0.4532 |
| 0.3 | 0.4317 |
| 0.4 | 0.4212 |
| 0.5 | 0.3978 |

| 0.6 | 0.3701 |
| 0.7 | 0.3382 |
| 0.8 | 0.3025 |
| 0.9 | 0.2635 |
| 1.0 | 0.2214 |

Calculating the sum of all values found and subtracting half the sum of the two extreme values (see formula (1)), multiply the result by the constant $\frac{3}{8\pi}$, and then on 0.1.

As a result, we get $P(G1) = 0.04525$, whence

$$P(G1' \cup G1'') = P(G1 \cup G1' \cup G1'') - P(G1) = 0.04753 - 0.04525 = 0.00228.$$

Concerning $P(G1')$, where

$$G1' = \{(x,y)|x \in [\sqrt{3}; 2], y \in [0; 1], x + y(2 - \sqrt{3}) \leq 2\},$$

similarly, to the previous, we receive (see table 6).

**Table 6.** The value of the corresponding integrals.

| $p1$ | $\displaystyle\int_{\sqrt{3}}^{2+p1(\sqrt{3}-2)} \left(2 - \sqrt{x^2 + p1^2}\right) dx$ |
| --- | --- |
| 0 | 0.0359 |
| 0.1 | 0.03489 |
| 0.2 | 0.03214 |
| 0.3 | 0.02807 |
| 0.4 | 0.02314 |
| 0.5 | 0.01778 |
| 0.6 | 0.01246 |
| 0.7 | 0.007597 |
| 0.8 | 0.003631 |
| 0.9 | 0.0009693 |
| 1.0 | 0.0000… |

$$P(G1') = 0.002132, \text{ whence}$$

$$P(G1'') = P(G1' \cup G1'') - P(G1') = 0.00228 - 0.002132 = 0.000148.$$

Given the symmetry $G2'$ and $G2''$ with respect to $G1'$ and $G1''$ relative to the bisector of each coordinate angle, we get:

$$P(G2') = P(G1') = 0.002132,$$

$$P(G2'') = P(G1'') = 0.000148.$$

Taking into account the symmetry with respect to the coordinate axes and with respect to the bisectors of the first and second coordinate angles, it is possible to calculate the probabilities of hitting any subsets of the set under consideration.

$$\Omega = \{(x; y)|x^2 + y^2 = 2^2\},$$

composed of an arbitrary set of subsets $G\emptyset, G1, G1', G1'', G2, G2', G2'', G3, G3'$ and their symmetric mappings with respect to the coordinate axes or bisectors of the second coordinate angle, at a given probability density.

Recall that together with the calculation of the probabilities of getting into different two-dimensional subsets of a two-dimensional set $\Omega$ in geometric interpretation the volumes of spatial bodies which

surfaces are described through the set functions of a kind are calculated $z = f(x, y)$, where $f(x, y)$ an essential function, by which describes the density probability distribution on the set $\Omega$.

Similarly, calculated probability of falling in convex sets on the plane with a normal two-dimensional probability distribution with the scattering center at a point $(0,0)$ and dispersions along the axes $Ox$ and $Oy$ equal $D_1 = \frac{1}{2}, D_2 = \frac{1}{2}$, the density of which is given in the form $f(x, y) = \frac{1}{\pi} e^{-(x^2+y^2)}$, $(x, y) \in R^2$, example

$$\iint_G f(x, y) dx dy,$$

where $G = \{(x, y) | (x - 1)^2 + y^2 \leq 1\}$, $G = \{(x, y) || x + y| \leq 1, |x - y| \leq 1\}$ etc.

## 4. Conclusions

Analyzing the above, we can conclude that the use of modern information and communication technologies, in particular the program GRAN1, is possible during conducting mathematics lessons, and the choice of means depends not only on the technical support of the educational process in a particular educational institution, but also on the level of teacher and student training to the use of information and communication technologies. The presence of an Internet connection in the educational institution makes it possible, through the use of cloud technologies, to provide the educational process with freely distributable software, including software Services from the package GRAN (which include GRAN1, GRAN2D, GRAN3D) with observance compliance with licensing agreements under applicable law.

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