Computational Problems in Forecasting in Complex Dynamical Systems

Bohdan Melnyk, Petro Stakhiv, Nataliya Melnyk and Stepan Trokhaniak

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Bohdan Melnyk
dept. Information Systems of Management
Ivan Franko National University of Lviv
Lviv, Ukraine
bohdan.melnyk@lnu.edu.ua

Petro Stakhiv
dept. Theoretical and General Electrical Engineering
Lviv Polytechnic National University
Lviv, Ukraine
petro.h.stakhiv@lpnu.ua

Nataliya Melnyk
dept. Information Systems of Management
Ivan Franko National University of Lviv
Lviv, Ukraine
nataliia.melnyk@lnu.edu.ua

Stepan Trokhaniak
dept. Information Systems of Management
Ivan Franko National University of Lviv
Lviv, Ukraine
stepan.trokhaniak@lnu.edu.ua

***Abstract*—The article considers some problems that arise when modeling complex dynamic systems, on the example of the distribution of the computer equipment market. The classic dynamic models used in electrical engineering and electronics are chosen for modeling. It is suggested to use optimization procedures to solve the problems that arise. They make it possible to increase the accuracy of the forecast obtained with the help of a dynamic model. Examples of solving specific optimization problems are considered. Recommendations are given for the application of the proposed approaches for the electrical industry, in particular, the electricity market.**

Keywords—dynamic model, optimization procedures, forecast, computer equipment market

I. Introduction

The concept of Industry 4.0 provides for the development and implementation of intelligent systems in all areas of human life: energy, communications, medicine, science, life, ets. The technical basis for such systems is a variety of computer equipment. It is obvious that the development of this base is closely connected with the existence of a competitive computer equipment market. The larger the market and the more diverse the range of equipment it offers, the faster and more efficient the implementation of computer systems.

Of course, the development of the computer equipment market requires investment. However, before investing their money in a particular business, the investor conducts a comprehensive market research. Such research should be based on scientifically sound methods of modeling and forecasting. In this article, we propose approaches to forecasting the distribution of the computer equipment market in Ukraine.

In Ukraine, the computer equipment market occupies more than 80% of the total domestic market of information technology [1]. Therefore, it is important for the national economy. There are many different sellers in this market. But most of them fall into one of four categories: consumer electronics nets (CEN), specialized computer stores (SCS), mobile communication stores (MCS) and B2B-sector enterprises (B2B) [2]. The computer equipment market is divided into a number of segments: personal computers (PC), laptops, displays, multifunction devices (MFD), etc. Each category of sellers determines its priorities of presence in a particular segment, which correspond to its trade policy in this market.

We propose to consider the computer equipment market as a complex dynamic system. To model its distribution, it is advisable to use dynamic models, which are borrowed from systems theory [3]. They are often called macromodels [4] and are used in electrical engineering and related fields. In this article, we will look at the challenges of using these models to predict the distribution of the computer equipment market. The proposed approaches to solving these problems can be adapted to the tasks that exist, in particular, in electrical engineering and electronics.

II. General Approaches to Modeling the Distribution of the Computer Equipment Market

The distribution of the computer equipment market over time is changing. To modeling the dynamic of the shares of categories of sellers in different segments of the market, it is proposed [3] to use a classical discrete dynamic model in form:

$$\begin{cases} \vec{x}^{(k+1)} = F \cdot \vec{x}^{(k)} + G \cdot \vec{v}^{(k)} \\ \vec{y}^{(k+1)} = C \cdot \vec{x}^{(k+1)}, k = 0,1,2,\ldots \end{cases}, \quad (1)$$

where $\vec{x}^{(k)}$ – vector of state variables, which characterize the change in the formal state of the market; $\vec{v}^{(k)}$ – vector of input variables that reflect the influence of external factors on the market; $\vec{y}^{(k)}$ – a vector of output variables that reflects the share of market segment held by the category of sellers; k – the moment of time in which the values of the components of the vectors are determined; F, G, C – matrix of formal parameters of the model.

The advantage of this model is that it simultaneously reproduces the dynamics of distribution in all market segments between all categories of sellers. The procedure for its construction does not depend on the number of market segments or the number of categories of sellers.

To identify the parameters of the model (1) are used known in the theory of dynamic systems algorithm of Ho-Kalman [4]. The software implementation of this algorithm in MathCad is presented in [3].

The model in the form (1) can be used to predict the dynamics of distribution of market segments between the categories of sellers, when none of the categories leaves any segment. However, an option is possible when at a certain point in time t_{out} some category ceases to trade a separate type of computer equipment. This means that it ceases to be present in the corresponding market segment. And this market segment will be divided between the categories which continue to sell this particular type of equipment. In order to take into account such changes in the market structure, proposed [3] to use a modified model as:

$$\begin{cases} \vec{x}^{(k+1)} = F \cdot \vec{x}^{(k)} + G \cdot \vec{v}^{(k)} \\ \vec{y}^{(k+1)} = T \cdot C \cdot \vec{x}^{(k+1)}, k = 0,1,2,\ldots \end{cases}, \quad (2)$$

where T is a diagonal matrix that has the form:

$$T = \begin{pmatrix} f(t_1) & 0 & \cdots & 0 \\ 0 & f(t_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & f(t_n) \end{pmatrix}, \quad (3)$$

where is the switch function $f(t_j) = \begin{cases} 1, t_j < t_{out} \\ 0, t_j \geq t_{out} \end{cases}$, $j = \overline{1,n}$, n - number of vector components $\vec{y}^{(k)}$ in the model (1).

III. PROBLEMS FOR PREDICTING THE DISTRIBUTION OF MARKET SEGMENTS BETWEEN SELLERS' CATEGORIES

Ho-Kalman algorithm allows you to build a model (1) that accurately reproduces the preliminary distribution of the market. However, when predicting the future distribution, this model gives certain errors. That is why there is a problem of adaptation of the model (1) in order to obtain adequate forecasts.

We are offered such an approach to solving this problem. First we find the forecast of distribution of market segments using the model (1). This forecast is considered as a zero approximation of the final result. Then we analyze the inadequacy of this zero approximation. Based on the analysis, we determine the criteria for achieving an adequate forecast. Taking into account they formulate an appropriate optimization task. At the last stage we solve this task. Its solution will be an adequate result of prediction.

During the use of the model (2) there is another problem. The projected shares of those categories of sellers who remain in a certain segment of the market do not take into account the loss of share by the category that leaves this segment. Obviously, the share lost will be distributed between the categories that remain. To predict this distribution, we propose to apply a separate optimization procedure.

IV. EXAMPLES AND RESULTS OF THE APPLICATION OF OPTIMIZATION PROCEDURES

Consider the example of distribution between categories of sellers of four segments of the computer equipment market ($j = \overline{1,4}$): PC ($j = 1$), laptops ($j = 2$), displays ($j = 3$), MFD ($j = 4$). Categories of sellers are numbered: CEN ($i = 1$), SCS ($i = 2$), MCS ($i = 3$), B2B ($i = 4$).

To construct a basic model (1) we use statistical data for four periods ($k = \overline{1,4}$) [3]. With the help of the Ho-Kalman algorithm, the corresponding parameters for this model were identified.

A. Optimization Procedures for Model (1)

Apply the resulting baseline (1) with appropriate parameters to predict the distribution of market segments on the 5th period ($k = 5$).The forecast result is presented in Table 1.

TABLE I. FORECAST OF DISTRIBUTION OF MARKET SEGMENTS IN THE 5TH PERIOD

Categories		**Share in market segments, %**			
Number	***Name***	***PC***	***laptops***	***displays***	***MFD***
1	CEN	34,7	58,0	38,4	56,1
2	SCS	8,2	21,0	23,8	21,2
3	MCS	-1,5	10,7	1,6	5,4
4	B2B	58,6	10,3	36,2	17,3

As can be seen from the table, in the PC segment, the share of MCS matters less than zero that is nonsense. Therefore, the forecast received for all categories w_i^0 ($i = \overline{1,4}$) will be considered a zero approximation for the final forecast w_i for this segment.

Obviously, the sum of the shares of all categories reaches a maximum at 100%. That is, the objective function for an optimization task has the form

$$\sum_{i=1}^{4} w_i \to max = 100 . \quad (4)$$

The share of any category can not be smaller zero, that is

$$w_i \geq 0, \; i = \overline{1,4} . \quad (5)$$

In order to the optimization procedure did not distort the prediction result, which is obtained using a model (1), it is necessary to guarantee the accuracy of this model in a certain neighborhood. To this end, introduce such restrictions for all w_i ($i = \overline{1,4}$)

$$\left| \frac{w_i - w_i^0}{w_i^0} \right| \leq \varepsilon_i , \quad (6)$$

where $0 \leq \varepsilon_i \leq 1$ are a predetermined errors.

Based on (4) - (6) we formulate an optimization task in general:

$$\begin{cases} w_1 + w_2 + w_3 + w_4 \to max \\ \sum_{i=1}^{4} w_i = 100 \\ \left| \frac{w_i - w_i^0}{w_i^0} \right| \le \varepsilon_i \\ w_i \ge 0 \\ 0 \le \varepsilon_i \le 1 \\ i = \overline{1,4} \end{cases} . \quad (7)$$

The solutions of this task are the final forecast for the 5th period of shares of categories of sellers $\tilde{w}_i$ ($i = \overline{1,4}$) in the PC segment. The forecast that is calculated using the conjugate gradient method is presented in Table 2.

TABLE II. FINAL FORECAST OF THE DISTRIBUTION OF THE PC SEGMENT

Categories	Share in the PC segment, %	Error
CEN	34,0	0,016
SCS	7,8	0,05
MCS	0,1	1,0
B2B	58,1	0,009

As can be seen from Table 2, the magnitude of the predicted MCS share is greater than zero. Comparison of data presented in Tables 1 and 2 shows that the optimization procedure did not distort the distribution trends, which is determined by model (1).

B. Optimization Procedures for Model (2)

Let in the 5th period ($k = 5 = t_{out}$) category MCS refused to trade of displays ($j = 3$). Consequently, when $k = 5$ we have $t_3 = t_{out}$ and accordingly, the switch function $f(t_3) = 0$. For all other $j \ne 3$ the switch function $f(t_j) = 1$. For the category MCS according to (3) matrix T for the model (2) will have the form

$$T = \begin{pmatrix} 1\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,1 \end{pmatrix} . \quad (8)$$

For other categories, T is identity matrix.

The Fig. 1 shows the distribution of the segment of the displays, after the use of the model (2) with the corresponding matrices T .

As can be seen from the chart, the category MCS has lost a share, which is equal to 1,6% in segment. Obviously, this share will share those categories that continue to trade displays. To determine the final distribution of the segment, we apply two methods: proportional method and optimization method.

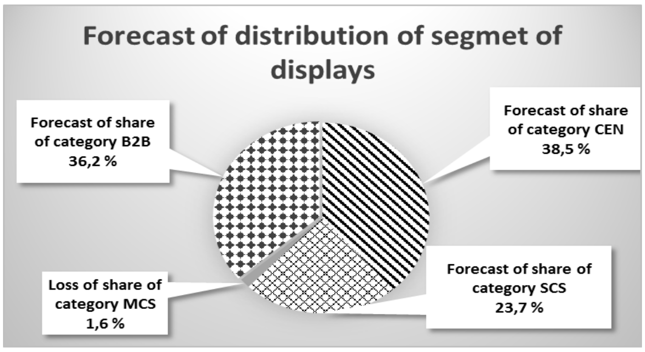

Fig. 1. Forecast of distribution of segment of the displays according to model (2)

1) Proportional method

Under normal conditions, the most likely is proportional distribution of the category MCS shares between categories CEN, SCS and B2B. That is, each category takes the part of the share of the category MCS, which is proportional to their shares in the segment. Namely, category CEN takes 38.5% of 1.6%, category SCS 23.7% and category B2B respectively 36.2%. The Fig. 2 shows the final distribution of the segment.

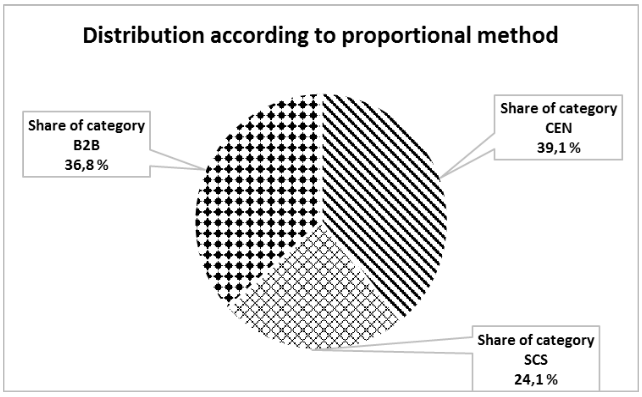

Fig. 2. Forecast of distribution of the segment of the displays by proportional method

2) Optimization method

In many cases, additional conditions must be taken into account when predicted. Therefore, the proportional method should be used at the initial stage of prediction. To take into account the additional conditions we will apply optimization procedures.

a) Optimization procedure 1

Let η_i^0 ($i = 1,2,4$) is the share of the i -th category according to the forecast received by model (2). As a result of optimization procedure, we obtain a share η_i . Obviously, $\eta_i \ge 0$ and the sum of shares of all categories should be equal to 100% ($\sum_{i=1,2,4} \eta_i = 100$). In addition, none of the categories (CEN, SCS and B2B) can additionally get more than a share which the category MCS has lost, that is 1,6% in the segment. Therefore, restrictions should be performed $\left| \eta_i - \eta_i^0 \right| \le 1{,}6\%$. Consequently, we have an optimization task in the form

$$\begin{cases} \eta_1 + \eta_2 + \eta_4 \to max \\ \sum_{i=1,2,4} \eta_i = 100 \\ \left|\eta_i - \eta_i^0\right| \le 1{,}6 \\ \eta_i \ge 0 \\ i = 1,2,4 \end{cases} . \quad (9)$$

The Fig. 3 shows the final forecast of distribution of the segment of the displays by this optimization procedure.

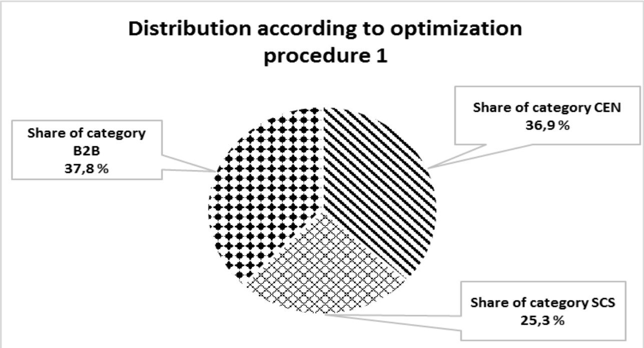

Fig. 3. Forecast of distribution of the segment of the displays by optimization procedure 1

b) Optimization procedure 2

Consider a more difficult situation. Let each of the categories (CEN, SCS and B2B) non plans to increase its share in the segment of the displays more than σ_i . Then in the task (9) must add restrictions $\left|\eta_i - \eta_i^0\right| \le \sigma_i$ ($i = 1,2,4$).

For example, consider a partial case. Let the category SCS ($i = 2$) not plan to increase its share in the segment of the displays more than 0,5%. For this case, the optimization task will have the form

$$\begin{cases} \eta_1 + \eta_2 + \eta_4 \to max \\ \sum_{i=1,2,4} \eta_i \to 100 \\ \left|\eta_i - \eta_i^0\right| \le 1{,}6 \\ \left|\eta_2 - \eta_2^0\right| \le 0{,}5 \\ \eta_i \ge 0 \\ i = 1,2,4 \end{cases} . \quad (10)$$

The Fig. 4 shows the final forecast of distribution of the segment of the displays by this optimization procedure.

For comparison in Table 3, the results of final forecasts received by different methods and procedures are summarized.

TABLE III. FINAL FORECASTS OF DISTRIBUTION OF THE SEGMENT OF THE DISPLAYS

Categories	Shares according to the final forecast, %		
	Proportional method	***Optimization procedure 1***	***Optimization procedure 2***
CEN	39,1	36,9	40,1
SCS	24,1	25,3	23,2
B2B	36,8	37,8	36,7

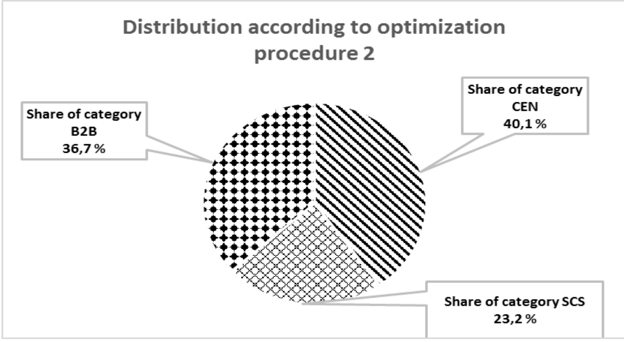

Fig. 4. Forecast of distribution of the segment of the displays by optimization procedure 2

Table 4 shows the difference between forecasts received by different methods and procedures.

TABLE IV. DIFFERENCE BETWEEN FORECASTS

Categories	The difference in predictions between methods, %		
	Proportional method - Optimization procedure 1	***Proportional method - Optimization procedure 2***	***Optimization procedure 1 - Optimization procedure 2***
CEN	2,2	-1,0	-3,2
SCS	-1,2	0,9	2,1
B2B	-1,0	0,1	1,1

Let us assume that proportional method specifies the tendency to distribute the share which the category MCS has lost. Based on the analysis of Table 4, it can be concluded that for prediction, the optimization procedure 2 is better than the optimization procedure 1. Smaller differences between predictions obtained by proportional method and according to optimization procedure 2 indicate that. Obviously, the result was influenced by additional restrictions which are in optimization task 2.

V. CONCLUSIONS

Dynamic models in the space of variable states, which are used in various fields, are effective for describing a system with a complex internal structure. However, in the case of constant parameters of such a model, the accuracy of predicting the behavior of the system in the future decreases. Appropriate optimization procedures must be applied to improve accuracy. They make it possible to reduce forecasting errors while maintaining the dynamic characteristics of the system described by the model.

The proposed optimization approaches can be applied in related electrical industries, for example, when forecasting the distribution of electricity production between producers using different generation sources, within a country or region.

REFERENCES

[1] Information Technology Industry in Ukraine. *Infographic atlas*. 2017. Retrieved from: URL: http://publications.chamber.ua/2018/IT/the-infographics-report-it-industry-of-ukraine-2017.pdf.

[2] GfK Ukraine. Retrieved from: http://www.gfk.ua

[3] N. Melnyk, M. Dyvak, P. Stakhiv, B. Melnyk, Z. Rihova, and M. Vohnoutova, "Modeling the Dynamics of Computer Hardware Market Distribution," in Advances in Intelligent Systems and Computing V. CSIT 2020, vol. 1293. Springer, 2021, pp. 823-840.

[4] P.H. Stakhiv, Y.Y. Kozak, and O.P. Hoholyuk, Discrete macromodeling in electrical engineering and related fields, Lviv: Polytechnic Publishing House, 2014 (in Ukrainian).