The New Standard of the Kilogram

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THE NEW STANDARD OF THE KILOGRAM

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Annotation

This paper proposes a new standard of the kilogram, the method of mass measurement. Measurements are carried on far from the Earth, which reduces the impact of the Earth gravitation. The mass is determined by the dynamic method (without measuring the force of attraction between the test bodies), using a laser interferometer.

Introduction

Currently, in the International System of Units (SI), the International Committee for Weights and Measures (CIPM - Comite' International des Poids et Mesures) defines the kilogram as the mass of international prototype Kilogram (IPK) [1], which is the Platinum-iridium cylinder.

The disadvantage of this standard of the kilogram IPK is that its mass varies with time [2]. It is now widely acknowledged that definition of the standard of the kilogram IPK should be revised.

Several new definitions of standard of the kilogram were proposed [3, 4], where certain physical constants have been offered to be fixed (Planck constant h, Avogadro number N_A) to form a new standard of the kilogram based on them.

Formulation of the problem

The kilogram, the standard of mass, is most commonly related to gravitational attraction force and inertia, that is why the new proposed standard of the kilogram is based on the law of gravitation and on the Newton's second law of motion [5]:

"The standard of the kilogram is the total mass of two balls (m_1+m_2)*, which get acceleration* $a=d^2R/dt^2$ *during their relative to each other free motion, equal* $a=\gamma\bullet(m_1+m_2)/R^2$*, where R – distance between balls centres,* γ *– gravitational constant, exactly equal* $6.67408\bullet10^{-11} m^3\ kg^{-1}\ c^{-2}$*"*

There are two different mass implicitly present in this definition of the kilogram: mass, participating in the law of attraction, gravitational mass and mass that participate in the Newton's second law, inert mass. The proposed standard of the kilogram postulates that these masses are equal.

Gravitational constant γ is the least accurately measured value [6]. Measurement of this constant is directly related to the standard of mass, and postulation of gravitational constant exact value $\gamma=6.67408\bullet10^{-11} m^3\ kg^{-1}\ c^{-2}$ in proposed standard of the kilogram solves this problem immediately. The specific value of gravitational constant will be clarified by the International Committee for Weights and Measures on the latest measurements of gravitational constant γ immediately before the acceptance of the standard of the kilogram. Each new standard of the kilogram offers a new method of mass measurement. In new standard of the kilogram, any dynamic method providing required accuracy may be used. Dynamic methods are simple to explicate and understand: two balls fall against each other under the force of gravity (figure 1), and there are not any other details in the mass measurement method (simplicity in understanding of standards for pupils and students is one of requirements of CIPM committee). Wherein dynamic methods are based on the most simple and accurate measured values (standards) of distance *meter* [7] (measurement of distance between mass R) and time *second* [8] (distance R variation in time).

Fig. 1. Gravitational attraction between two bodies.

The Earth with the mass M is used in dynamic method of mass measurement (fig. 1) [9]. Mass (M+m) can be accurately measured [10] under the third Kepler's law by placing the test mass m into elliptic orbit and measuring the parameters of orbit L (orbital period T and semi-major axis of orbit L):

$$\gamma \bullet T^2(M+m)=4\pi^2 \bullet L^3 \qquad (1)$$

Considering that M>>m, this method allows to measure the Earth mass, but not the test mass m. The Earth mass is much bigger than the test mass, that is why a determining factor for the acceleration of gravity g is the mass of the Earth. To eliminate this flaw, two masses m_1 and m_2 must be put into space in zero-gravity and their motion under attractive interaction must be measured. One of the possible methods of measuring mass is proposed in [11, 12].

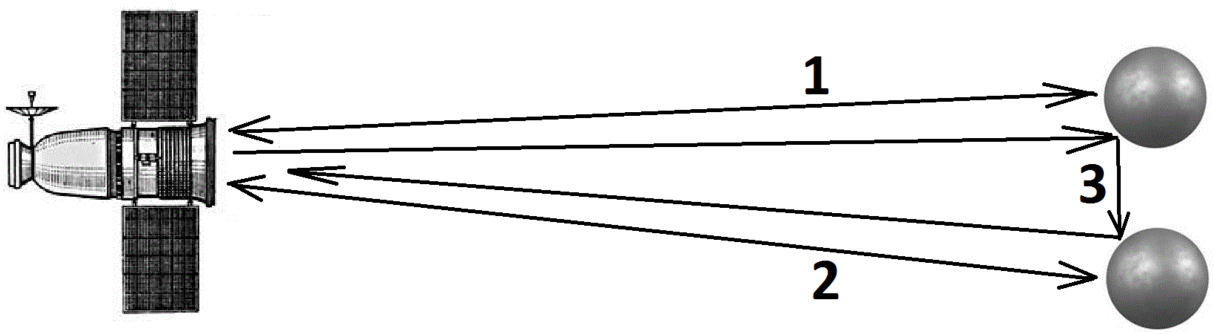

Fig. 2. Dynamic method of measurement of two balls' mass. 1, 2 – interference measurments of the distance between measuring satellite and two balls m_1 and m_2, 3 – interference measurments of the distance between balls.

Measurement of the sum of masses (m_1 + m_2) by the proposed method occurs as follows:

- Two balls with masses m_1 and m_2 are placed into zero-gravity at the distance R, which equals about several diameters of balls.
- Interference measurements of the distance between measuring satellite and two balls m_1 and m_2 (1,2 in fig. 2) are carried out from the measuring satellite located at a much larger distance from the balls (comparing with the distances between the balls R).
- Interference measurements of the distance between the balls (3 in Fig. 2) are performed with a laser rays double reflection of the measuring satellite: from the ball m_1 to the ball m_2, and then from the ball m_2 to the measuring satellite.
- After hanging the balls m_1 and m_2 begin a relative motion under the action of mutual gravitational attraction forces, where the relative acceleration of the balls ($a = d^2R / dt^2$, equal to $a = \gamma \cdot (m_1 + m_2) / R^2$) is proportional to the sum of the masses $(m_1 + m_2)$.
- Accordingly, measuring the relative motion of balls, using the data of interference measurements, we measure the sum of the masses $(m_1 + m_2)$.

Motion of two masses m_1 and m_2 under the action of mutual gravitational attraction forces is thoroughly studied (two body problem [5, 10, 13, 14]). The type of motion (orbits of balls m_1 and m_2 relatively to the center of two balls masses) depends on initial relative speed of the balls dR/dt, depends on integral sign of energy h:

$$h=(dR/dt)^2-2\gamma\cdot(m_1+m_2)/R \quad (2)$$

- At h<0 we get elliptic orbit,
- At h=0 we get parabolic orbit,
- At h>0 we get hyperbolic orbit.

At the initial relative velocity of the balls m_1 and m_2, which is directed along the straight line passing through the centers of the balls, we obtain a rectilinear trajectory of the balls motion.

The probability of obtaining a particular trajectory of the balls motion varies greatly: the probability of occurrence of parabolic and rectilinear trajectories is close to zero.

Elliptic orbit ($h<0$) is limited: the balls m_1 and m_2 will not fly apart to the infinity and will "eternally" rotate about the center of balls masses m_1 and m_2. Therefore, this orbit is preferable because it increases the observation time, the measurement time of the balls' trajectory. This orbit makes it possible to obtain and measure the total mass of balls m_1 and m_2 not only from acceleration measurements d^2R / dt^2 and the distance R between the balls, but also directly from the parameters of the elliptic orbit (1) (the balls circulation period T around the center of mass and the semi-major axis of the orbit L).

Results

Let's consider a simple example. Let the balls m_1 and m_2 be of copper, then for balls with diameters of 30cm we get the masses m_1 and m_2 equal to 126kg. Accordingly, we obtain an elliptic orbit ($h <0$) at the initial relative balls velocity dR/dt less than 0.18 mm/s (with the initial distance between the centers of the balls R = 1m). That is, obtaining a limited elliptical orbit of balls is associated with the need of getting a very small (less than 0.18 mm/s in our example) initial relative balls velocity when hanging, with initial placement into zero-gravity.

If orbit is taken as circular with the radius L=0,5m, then from (1) we get the period T of motion of the copper balls, with the diameters of 30 sm, about T=4,7 hours. That is, for bounded elliptic orbits of the balls from our example, we obtain a sufficiently large (for getting a large amount of measurement data), but quite adequate time for measuring one period T of balls orbital motion around the center of balls masses.

Lets consider the measuring frequency of the measuring satellite optical receiver (frame rate) necessary for measuring the interference fringe pattern. For the balls relative motion velocity of 0.18 mm/s and for the red color laser radiation of a measuring satellite with a wavelength of λ= 0.68 nm, and for ten measurements during the time of balls shifting motion at the red color wavelength λ, we obtain the frequency of optical receiver measurements about 2600 measurements per second.

The increase in the matter density of the balls, the convergence of the initial distance between the balls simplifies the carrying conditions of dynamic measurements, since it increases the gravitational interaction. If the balls are platinum, with diameters of 30cm and at the initial distance between the balls is 0.6m, we obtain an elliptic orbit (h <0) at the initial relative velocity dR/dt of balls less than 3.7mm/s, the period T of copper balls circular motion is about T=1.4 hours.

Thus, our simple example shows that the measurement of the total mass of two balls (m_1 + m_2) by a dynamic interference method in zero-gravity is quite avaliable at the current level of technology development. At the same time, these measurements are based on the simplest and most accurately measured values (standards) of distance and time, and great accuracy of mass measurement is attainable, so it is possible to create a new standard of mass on this principle.

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