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Local and $2-$local $\frac{1}{2}-$derivations of octonion and Okubo algebras

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***Abstract*— In this work, we introduce the notion of local and 2-local $\frac{1}{2}$-derivations and describe local and 2-local $\frac{1}{2}$-derivation of the octonion and Okubo algebras. We prove that every local and 2-local $\frac{1}{2}$-derivation on the octonion and Okubo algebras is a $\frac{1}{2}$-derivation.**

Keywords*— *Octonion algebra; Okubo algebra; $\frac{1}{2}$-derivation; local $\frac{1}{2}$-derivation, 2-local $\frac{1}{2}$-derivation

I. INTRODUCTION

Filippov studied δ-derivations of Lie algebras in a series of papers [13-15]. The space of δ-derivations includes usual derivations ($\delta = 1$), anti-derivations ($\delta = -1$) and elements from the centroid. In [14] it was proved that prime Lie algebras, as a rule, do not have nonzero δ-derivations (provided $\delta \neq 1, -1, 0, \frac{1}{2}$), and all $\frac{1}{2}$-derivations of an arbitrary prime Lie algebra A over the field $\mathbb{F}$ of characteristic $p \neq 2, 3$ with a non-degenerate symmetric invariant bilinear form were described. It was proved that if A is a central simple Lie algebra over a field of characteristic $p \neq 2, 3$ with a non-degenerate symmetric invariant bilinear form, then any $\frac{1}{2}$-derivation φ has the form $\varphi = \lambda x$ for some $\lambda \in \mathbb{F}$.

In recent decades, a well-known and active direction in the study of derivations of associative algebras and rings is the problem about local derivations. The notion of local derivation on algebras was introduced by R.V. Kadison [16], D.R. Larson and A.R. Sourour [19]. A local derivation on an algebra A is a linear map $\Delta : A \to A$ which satisfies that for any $x \in A$, there exists a derivation $D_x : A \to A$ (depending on x) such that $\Delta(x) = D_x(x)$. The main problems concerning local derivations are to find conditions under which local derivations become derivations and to present examples of algebras with local derivations that are not derivations. Several authors investigated local derivations for finite or infinite dimensional Lie algebras [1-9, 11, 17, 18, 20, 21], and it was proved that every local derivation on many Lie algebras (for examples, semi-simple Lie algebras, Borel subalgebras of finite-dimensional simple Lie algebras, the Schrödinger algebra s_n in $n+1$-dimensional space-time) is a derivation.

Investigation of local and 2-local δ-derivations on Lie algebras was initiated in [18] by A. Khudoyberdiyev and B. Yusupov. Namely, in [18] it is proved we introduce the notion of local and 2-local δ-derivations and describe local and 2-local $\frac{1}{2}$-derivation of finite-dimensional solvable Lie algebras with filiform, Heisenberg, abelian nilradicals. Moreover, in the work [18] is given the description of local $\frac{1}{2}$-derivation of oscillator Lie algebras, conformal perfect Lie algebras, and Schrödinger algebras. B.Yusupov, V.Vaisova and T. Madrakhimov proved similar results concerning local $\frac{1}{2}$-derivations of naturally graded quasi-filiform Leibniz algebras of type I in their recent paper [21]. They proved that quasi-filiform Leibniz algebras of type I, as a rule, admit local $\frac{1}{2}$-derivations which are not $\frac{1}{2}$-derivations.

II. MAIN RESULTS

$\frac{1}{2}$-derivation of the octonion algebra $\mathcal{O}$.

Definition [10]. Let L be an arbitrary 2-torsion free unital ring. The octonion algebra (denoted by $\mathcal{O}$) over L is a class of non-associative algebra. It is a unital non-associative algebra of dimension 8 with the basis $\mathcal{B}=\{e_0,e_1,e_2,e_3,e_4,e_5,e_6,e_7\}$ and the product defined in the following relation:

$$e_i\cdot e_j=\begin{cases} e_j, & if\ i=0;\\ e_i, & if\ j=0;\\ -\delta_{ij}e_0+\epsilon_{ijk}e_k, & \text{otherwise,}\end{cases}$$

where δ_{ij} is Kronecker delta and ϵ_{ijk} is a completely antisymmetric tensor with value $+1$ when $ijk=123,145,176,246,257,347,365$.

Theorem. Let $\mathcal{O}$ Octonion algebra over a field of characteristic zero. Then any $\frac{1}{2}$-derivation of octonion algebra has the following form:

$$D(e_i)=\beta_{11}e_i,\ 0\le i\le 7$$

Proof: Let D be $\frac{1}{2}$-derivation of $\mathcal{O}$. Then we write

$$D(e_{i-1})=\sum_{j=1}^{8}\beta_{ij}e_{j-1},\ 1\le i\le 8.$$

Applying D on the $e_{i-1}\cdot e_{i-1}$:

$$D(e_{i-1}\cdot e_{i-1})=\frac{1}{2}\left(\sum_{j=1}^{8}\beta_{ij}e_{j-1}e_{i-1}+e_{i-1}\sum_{j=1}^{8}\beta_{ij}e_{j-1}\right)=$$

$$=\beta_{i1}e_{i-1}-\beta_{ii}e_0=-D(e_0)=-\sum_{j=1}^{8}\beta_{1j}e_{j-1}$$

By comparing the coefficients of basis vectors, we get:

$\beta_{11}=\beta_{ii},\beta_{i1}=-\beta_{1i},\ 2\le i\le 8,\beta_{1j}=0,\ j\ne i,2\le j\le 8$.

By using these relations, we find that

$\beta_{1j}=\beta_{j1}=0,2\le j\le 8$.

Applying D on the product $e_0\cdot e_{k-1}$, where $2\le k\le 8$:

$$D(e_o\cdot e_{k-1})=D(e_{k-1})=\frac{1}{2}\left(\sum_{j=1}^{8}\beta_{1j}e_{j-1}\cdot e_{k-1}+\sum_{j=1}^{8}\beta_{kj}e_{j-1}\right)=$$

$$=\frac{1}{2}\left(\beta_{11}e_{k-1}+\sum_{j=1}^{8}\beta_{kj}e_{j-1}\right)=\sum_{j=1}^{8}\beta_{kj}e_{j-1}$$

By comparing the coefficients of basis vectors, we obtain: $\beta_{kj}=0,1\le j\le 8,j\ne k$.

We complete the proof of the theorem.

$\frac{1}{2}$-derivation of the Okubo algebra $\mathbb{O}$.

Assume first that the characteristic of $\mathbb{F}$ is not three, so $\mathbb{F}$ contains a primitive cubic root ω of 1.

Let $\mathbb{M}_3(\mathbb{F})$ the associative algebra of 3×3 matrices over $\mathbb{F}$. Let $\mathfrak{sl}_3(\mathbb{F})$ denote the corresponding special Lie algebra. Define a binary multiplication on $\mathfrak{sl}_3(\mathbb{F})$ as follows: $x*y=\omega xy-\omega^2 xy-\frac{\omega-\omega^2}{3}\mathrm{tr}(xy)1$. (1)

The vector space $\mathbb{O}=\mathfrak{sl}_3(\mathbb{F})$ endowed with the multiplication in (1) and nonsingular quadratic form $n(x)=sr(x)$ is a symmetric composition algebra: $n(x*y)=n(x)n(y), n(x*y,z)=n(x,y*z)$, for any x,y,z.

Definition[12]. The symmetric composition algebra $(\mathbb{O},*,n)$ is called the Okubo algebra over $\mathbb{F}(char\mathbb{F}\ne 3)$ **.**

Theorem: Let $\mathbb{O}$ Okubo algebras over a field of characteristic zero. Then any $\frac{1}{2}$-derivation of Okubo algebra has the following form:

$$\varphi(e_1)=\alpha_{11}e_1,\quad \varphi(e_2)=\alpha_{11}e_2,\quad \varphi(u_1)=\alpha_{11}u_1,$$
$$\varphi(u_2)=\alpha_{11}u_2,\quad \varphi(u_3)=\alpha_{11}u_3,\quad \varphi(v_1)=\alpha_{11}v_1,$$
$$\varphi(v_2)=\alpha_{11}v_2,\quad \varphi(v_3)=\alpha_{11}v_3.$$

Proof: Let φ be a $\frac{1}{2}$-derivation of Okubo algebra. Then we write:

$$\varphi(e_1)=\alpha_{11}e_1+\alpha_{12}e_2+\beta_{11}u_1+\beta_{12}u_2+\beta_{13}u_3+ +\delta_{11}v_1+\delta_{12}v_2+\delta_{13}v_3,$$
$$\varphi(e_2)=\alpha_{21}e_1+\alpha_{22}e_2+\beta_{21}u_1+\beta_{22}u_2+\beta_{23}u_3+ +\delta_{21}v_1+\delta_{22}v_2+\delta_{23}v_3,$$
$$\varphi(u_1)=\alpha_{31}e_1+\alpha_{32}e_2+\beta_{31}u_1+\beta_{32}u_2+\beta_{33}u_3+ +\delta_{31}v_1+\delta_{32}v_2+\delta_{33}v_3,$$
$$\varphi(u_2)=\alpha_{41}e_1+\alpha_{42}e_2+\beta_{41}u_1+\beta_{42}u_2+\beta_{43}u_3+ +\delta_{41}v_1+\delta_{42}v_2+\delta_{43}v_3,$$
$$\varphi(u_3)=\alpha_{51}e_1+\alpha_{52}e_2+\beta_{51}u_1+\beta_{52}u_2+\beta_{53}u_3+ +\delta_{51}v_1+\delta_{52}v_2+\delta_{53}v_3,$$
$$\varphi(v_1)=\alpha_{61}e_1+\alpha_{62}e_2+\beta_{61}u_1+\beta_{62}u_2+\beta_{63}u_3+ +\delta_{61}v_1+\delta_{62}v_2+\delta_{63}v_3,$$
$$\varphi(v_2)=\alpha_{71}e_1+\alpha_{72}e_2+\beta_{71}u_1+\beta_{72}u_2+\beta_{73}u_3+ +\delta_{71}v_1+\delta_{72}v_2+\delta_{73}v_3,$$
$$\varphi(v_3)=\alpha_{81}e_1+\alpha_{82}e_2+\beta_{81}u_1+\beta_{82}u_2+\beta_{83}u_3+ +\delta_{81}v_1+\delta_{82}v_2+\delta_{83}v_3.$$

We obtain the equalities below by applying φ the identities:

$$e_1e_1=e_2,\ v_1u_3=-e_2,\ v_2u_1=-e_2,\ v_3u_2=-e_2$$

(All relations between the coefficients of the basis vectors below are obtained due to the uniqueness of the representation of a vector $\varphi(x)$ in the basis $\{e_1,e_2,u_1,u_2,u_3,v_1,v_2,v_3\}$).

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$$\varphi(e_2)=\varphi(e_1e_1)=\frac{1}{2}\big((\alpha_{11}e_1+\alpha_{12}e_2+\beta_{11}u_1+\beta_{12}u_2+\beta_{13}u_3+\delta_{11}v_1+\delta_{12}v_2+\delta_{13}v_3)e_1+e_1(\alpha_{11}e_1+\alpha_{12}e_2+\beta_{11}u_1+\beta_{12}u_2+\beta_{13}u_3+\delta_{11}v_1+\delta_{12}v_2+\delta_{13}v_3)\big)=$$
$$=\frac{1}{2}\left(2\alpha_{11}e_2-\beta_{11}u_2-(\beta_{12}+\beta_{13})u_3-\delta_{12}v_2\right)$$

$$\alpha_{21}=0,\ \alpha_{22}=\alpha_{11},\ \beta_{21}=-\frac{1}{2}\beta_{13},\quad \beta_{22}=-\frac{1}{2}\beta_{11},\ \beta_{23}=-\frac{1}{2}\beta_{12},\ \delta_{21}=-\frac{1}{2}\delta_{12},\quad \delta_{22}=-\frac{1}{2}\delta_{13},\ \delta_{23}=0. \tag{2}$$

$$\varphi(e_2)=-\varphi(v_1u_3)=-\frac{1}{2}\big((\alpha_{61}e_1+\alpha_{62}e_2+\beta_{61}u_1+\beta_{62}u_2+\beta_{63}u_3+\delta_{61}v_1+\delta_{62}v_2+\delta_{63}v_3)u_3+v_1(\alpha_{51}e_1+\alpha_{52}e_2+\beta_{51}u_1+\beta_{52}u_2+\beta_{53}u_3+\delta_{51}v_1+\delta_{52}v_2+\delta_{53}v_3)\big)=$$
$$=-\frac{1}{2}\left(-(\beta_{53}+\delta_{61})e_2+\delta_{51}u_1-\alpha_{62}u_2-\delta_{52}u_3-\beta_{62}v_1-\alpha_{52}v_2+\beta_{63}v_3\right).$$

$$\alpha_{22}=\frac{1}{2}(\beta_{53}+\delta_{61}),\beta_{21}=-\frac{1}{2}\delta_{51},\quad \beta_{22}=\frac{1}{2}\alpha_{62},\ \delta_{22}=\frac{1}{2}\alpha_{52},\ \beta_{23}=\frac{1}{2}\delta_{52},\quad \delta_{21}=\frac{1}{2}\beta_{62},\ \delta_{23}=\frac{1}{2}\beta_{63}. \tag{3}$$

$$\varphi(e_2)=-\varphi(v_2u_1)=-\frac{1}{2}\big((\alpha_{71}e_1+\alpha_{72}e_2+\beta_{71}u_1+\beta_{72}u_2+\beta_{73}u_3+\delta_{71}v_1+\delta_{72}v_2+\delta_{73}v_3)u_1+v_2(\alpha_{31}e_1+\alpha_{32}e_2+\beta_{31}u_1+\beta_{32}u_2+\beta_{33}u_3+\delta_{31}v_1+\delta_{32}v_2+\delta_{33}v_3)\big)=$$
$$=-\frac{1}{2}\left(-(\beta_{31}+\delta_{72})e_2+\delta_{33}u_1+\delta_{32}u_2-\alpha_{72}u_3+\beta_{71}v_1-\beta_{73}v_2-\alpha_{32}v_3\right).$$

$$\alpha_{22}=\frac{1}{2}(\beta_{31}+\delta_{72}),\beta_{21}=\frac{1}{2}\delta_{33},\quad \beta_{22}=-\frac{1}{2}\delta_{32},\ \delta_{22}=\frac{1}{2}\beta_{73},\ \beta_{23}=\frac{1}{2}\alpha_{72},\quad \delta_{21}=-\frac{1}{2}\beta_{71},\ \delta_{23}=\frac{1}{2}\alpha_{32}. \tag{4}$$

$$\varphi(e_2)=-\varphi(v_3u_2)=-\frac{1}{2}\big((\alpha_{81}e_1+\alpha_{82}e_2+\beta_{81}u_1+\beta_{82}u_2+\beta_{83}u_3+\delta_{81}v_1+\delta_{82}v_2+\delta_{83}v_3)u_2+v_3(\alpha_{41}e_1+\alpha_{42}e_2+\beta_{41}u_1+\beta_{42}u_2+\beta_{43}u_3+\delta_{41}v_1+\delta_{42}v_2+\delta_{43}v_3)\big)=$$
$$=-\frac{1}{2}\left(-(\beta_{42}+\delta_{83})e_2-\alpha_{82}u_1-\delta_{41}u_2-\alpha_{42}v_1+\beta_{82}v_2-\beta_{81}v_3+\delta_{43}u_3\right).$$

$$\alpha_{22}=\frac{1}{2}(\beta_{42}+\delta_{83}),\beta_{21}=\frac{1}{2}\alpha_{82},\quad \beta_{22}=\frac{1}{2}\delta_{41},\ \delta_{22}=-\frac{1}{2}\beta_{82},\ \beta_{23}=-\frac{1}{2}\delta_{43},\quad \delta_{21}=\frac{1}{2}\alpha_{42},\ \delta_{23}=\frac{1}{2}\beta_{81}. \tag{5}$$

Now applying φ on identities:

$e_2e_2=e_1,\ u_1v_3=-e_1,\ u_2v_1=-e_1,\ u_3v_2=-e_1$.

$$\varphi(e_1)=\varphi(e_2e_2)=\frac{1}{2}\big((\alpha_{21}e_1+\alpha_{22}e_2+\beta_{21}u_1+\beta_{22}u_2+\beta_{23}u_3)+\delta_{21}v_1+\delta_{22}v_2+\delta_{23}v_3)e_2+e_1(\alpha_{21}e_1+\alpha_{22}e_2+\beta_{21}u_1+\beta_{22}u_2+\beta_{23}u_3+\delta_{21}v_1+\delta_{22}v_2+\delta_{23}v_3)\big)=$$
$$=\frac{1}{2}\left(2\alpha_{22}e_1-\beta_{22}u_1-\beta_{23}u_2-\beta_{21}u_3-\delta_{23}v_1-\delta_{21}v_2-\delta_{22}v_3\right).$$

$$\alpha_{11}=\alpha_{22},\alpha_{12}=0,\ \beta_{11}=-\frac{1}{2}\beta_{22},\quad \beta_{12}=-\frac{1}{2}\beta_{23},\ \beta_{13}=-\frac{1}{2}\beta_{21},\ \delta_{11}=-\frac{1}{2}\delta_{23},\quad \delta_{12}=-\frac{1}{2}\delta_{21},\ \delta_{13}=-\frac{1}{2}\delta_{22}. \tag{6}$$

$$\varphi(e_1)=-\varphi(u_1v_3)=-\frac{1}{2}\big((\alpha_{31}e_1+\alpha_{32}e_2+\beta_{31}u_1+\beta_{32}u_2+\beta_{33}u_3+\delta_{31}v_1+\delta_{32}v_2+\delta_{33}v_3)v_3+u_1(\alpha_{81}e_1+\alpha_{82}e_2+\beta_{81}u_1+\beta_{82}u_2+\beta_{83}u_3+\delta_{81}v_1+\delta_{82}v_2+\delta_{83}v_3)\big)=-\frac{1}{2}\left(-(\beta_{31}+\delta_{83})e_1-\delta_{32}u_1-\alpha_{81}u_2+\delta_{33}u_3+\beta_{81}v_1-\alpha_{31}v_2-\beta_{82}v_3\right).$$

$$\alpha_{11}=\frac{1}{2}(\beta_{31}+\delta_{83}),\alpha_{12}=0,\ \beta_{11}=\frac{1}{2}\delta_{32},\quad \beta_{12}=\frac{1}{2}\alpha_{81},\ \beta_{13}=\frac{1}{2}\delta_{33},\ \delta_{11}=-\frac{1}{2}\beta_{81},\quad \delta_{12}=\frac{1}{2}\alpha_{31},\ \delta_{13}=\frac{1}{2}\beta_{82}. \tag{7}$$

$$\varphi(e_1)=-\varphi(u_2v_1)=-\frac{1}{2}\big((\alpha_{41}e_1+\alpha_{42}e_2+\beta_{41}u_1+\beta_{42}u_2+\beta_{43}u_3+\delta_{41}v_1+\delta_{42}v_2+\delta_{43}v_3)v_1+u_2(\alpha_{61}e_1+\alpha_{62}e_2+\beta_{61}u_1+\beta_{62}u_2+\beta_{63}u_3+\delta_{61}v_1+\delta_{62}v_2+\delta_{63}v_3)\big)=-\frac{1}{2}\left(-(\beta_{42}+\delta_{62})e_2+\delta_{41}u_1-\delta_{43}u_2-\beta_{63}v_1+\beta_{62}v_2-\alpha_{41}v_3-\alpha_{61}u_3\right)$$

$$\alpha_{11}=\frac{1}{2}(\beta_{42}+\delta_{61}),\beta_{12}=\frac{1}{2}\delta_{43},\ \alpha_{12}=0,\quad \delta_{11}=\frac{1}{2}\beta_{63},\ \beta_{11}=-\frac{1}{2}\delta_{41},\ \beta_{13}=\frac{1}{2}\alpha_{61},\quad \delta_{12}=-\frac{1}{2}\beta_{62},\ \delta_{13}=\frac{1}{2}\alpha_{41}. \tag{8}$$

$$\varphi(e_1) = -\varphi(u_3v_2) = -\frac{1}{2}\big((\alpha_{51}e_1 + \alpha_{52}e_2 + \beta_{51}u_1 + \beta_{52}u_2 +$$
$$+\beta_{53}u_3 + \delta_{51}v_1 + \delta_{52}v_2 + \delta_{53}v_3)v_2 + u_3(\alpha_{71}e_1 + \alpha_{72}e_2 +$$
$$+\beta_{71}u_1 + \beta_{72}u_2 + \beta_{73}u_3 + \delta_{71}v_1 + \delta_{72}v_2 + \delta_{73}v_3)\big) =$$
$$= -\frac{1}{2}\big(-(\beta_{53} + \delta_{72})e_1 - \alpha_{71}u_1 + \delta_{52}u_2 -$$
$$-\alpha_{51}v_1 - \beta_{71}v_2 + \beta_{73}v_3 - \delta_{51}u_3\big).$$

$$\alpha_{11} = \frac{1}{2}(\beta_{53} + \delta_{72}), \beta_{12} = -\frac{1}{2}\delta_{52},$$
$$\alpha_{12} = 0,\ \beta_{11} = \frac{1}{2}\alpha_{71},\ \beta_{13} = \frac{1}{2}\delta_{51}, \quad (9)$$
$$\delta_{11} = \frac{1}{2}\alpha_{51},\ \delta_{13} = -\frac{1}{2}\beta_{73}.$$

from the relations (2)- (9) we obtain:

$$\alpha_{21} = 0, \quad \alpha_{22} = \alpha_{11},$$
$$\delta_{61} = \delta_{72} = \delta_{83},$$
$$\beta_{53} = \beta_{42} = \beta_{31},$$
$$\beta_{21} = \beta_{13} = \delta_{51} = \delta_{33} = \alpha_{82} = 0,$$
$$\beta_{22} = \beta_{11} = \alpha_{62} = \delta_{32} = \delta_{41} = 0,$$
$$\beta_{23} = \beta_{12} = \delta_{52} = \alpha_{72} = \delta_{43} = 0,$$
$$\delta_{21} = \delta_{12} = \beta_{62} = \beta_{71} = \alpha_{42} = 0,$$
$$\delta_{22} = \delta_{13} = \alpha_{52} = \beta_{73} = \beta_{82} = 0,$$
$$\delta_{23} = \beta_{63} = \alpha_{32} = \beta_{81} = \delta_{11} = 0,$$
$$\alpha_{81} = \delta_{43} = \beta_{62} = \beta_{63} = \alpha_{61} = 0,$$
$$\alpha_{31} = \alpha_{32} = \alpha_{41} = \alpha_{51} = \alpha_{71} = 0.$$

By using these relations, we write:

$$\varphi(e_1) = \alpha_{11}e_1, \qquad \varphi(e_2) = \alpha_{11}e_2,$$
$$\varphi(u_1) = \beta_{31}u_1 + \beta_{32}u_2 + \beta_{33}u_3 + \delta_{31}v_1,$$
$$\varphi(u_2) = \beta_{41}u_1 + \beta_{42}u_2 + \beta_{43}u_3 + \delta_{42}v_2.$$

Next, we will obtain equalities below by applying φ on identities:

$$e_1u_1 = 0, \quad u_1e_2 = 0, \quad u_2u_1 = 0, \quad u_2e_2 = 0,$$
$$e_2u_2 = -u_1,\ u_3e_2 = 0, \quad u_3e_1 = -u_1,\ v_1u_1 = 0,$$
$$v_1v_1 = u_1, \quad v_2v_1 = 0, \quad v_3v_2 = 0.$$

$$\varphi(e_1u_1) = \frac{1}{2}\big(\alpha_{11}e_1u_1 + e_1(\beta_{31}u_1 + \beta_{32}u_2 + \beta_{33}u_3 + \delta_{31}v_1)\big) =$$
$$= \frac{1}{2}(-\delta_{31}v_3) = 0$$
$$\varphi(u_1e_2) = \frac{1}{2}\big((\beta_{31}u_1 + \beta_{32}u_2 + \beta_{33}u_3 + \delta_{31}v_1)e_2 + \alpha_{11}u_1e_2\big) =$$
$$= -\frac{1}{2}\delta_{31}v_1 = 0$$
$$\varphi(u_2u_1) = \frac{1}{2}\big((\beta_{41}u_1 + \beta_{42}u_2 + \beta_{43}u_3 + \delta_{42}v_2)u_1 +$$
$$+u_2(\beta_{31}u_1 + \beta_{32}u_2 + \beta_{33}u_3 + \delta_{31}v_1)\big) =$$
$$= \frac{1}{2}(\beta_{41}v_1 - \beta_{43}v_2 - \delta_{42}e_2 + \beta_{32}v_2 - \beta_{33}v_1 - \delta_{31}e_1) = 0$$

$$\varphi(e_2u_2) = \frac{1}{2}\big(\alpha_{11}e_2u_2 + e_2(\beta_{41}u_1 + \beta_{42}u_2 + \beta_{43}u_3 + \delta_{42}v_2)\big) =$$
$$= \frac{1}{2}(-\alpha_{11}u_1 - \beta_{41}u_3 - \beta_{42}u_1 - \beta_{43}u_2) = -\varphi(u_1)$$
$$\varphi(u_3e_2) = \frac{1}{2}\big((\beta_{51}u_1 + \beta_{52}u_2 + \beta_{53}u_3 + \delta_{53}v_3)e_2 + \alpha_{11}u_3e_2\big) =$$
$$= -\frac{1}{2}\delta_{53}v_1 = 0$$
$$\varphi(u_3e_1) = \frac{1}{2}\big((\beta_{51}u_1 + \beta_{52}u_2 + \beta_{53}u_3 + \delta_{53}v_3)e_1 + \alpha_{11}u_3e_1\big) =$$
$$\frac{1}{2}(-\beta_{51}u_2 - \beta_{52}u_3 - \beta_{53}u_1 - \alpha_{11}u_1) = -\varphi(u_1)$$
$$\varphi(v_1u_1) = \frac{1}{2}\big((\beta_{61}u_1 + \delta_{61}v_1 + \delta_{62}v_2 + \delta_{63}v_3)u_1 + \alpha_{11}v_1u_1\big) =$$
$$= \frac{1}{2}(\beta_{61}v_1 - \delta_{62}e_2) = 0$$
$$\varphi(v_1v_1) = \frac{1}{2}\big((\delta_{61}v_1 + \delta_{63}v_3)v_1 + v_1(\delta_{61}v_1 + \delta_{63}v_3)\big) =$$
$$= \frac{1}{2}(2\delta_{61}u_1 - \delta_{63}u_2) = \varphi(u_1)$$
$$\varphi(v_2v_1) = \frac{1}{2}\big((\beta_{72}u_2 + \delta_{71}v_1 + \alpha_{11}v_2 + \delta_{73}v_3)v_1 + \alpha_{11}v_2v_1\big) =$$
$$= \frac{1}{2}(-\beta_{72}e_1 + \delta_{71}u_1 - \delta_{73}u_2) = 0$$
$$\varphi(v_3v_2) = \frac{1}{2}\big((\beta_{83}u_3 + \delta_{81}v_1 + \delta_{82}v_2 + \alpha_{11}v_3)v_2 + \alpha_{11}v_3v_2\big) =$$
$$= \frac{1}{2}(-\beta_{83}e_1 - \delta_{81}u_3 + \delta_{82}u_2) = 0$$

From the equalities above, we derive the following relations:

$$\beta_{52} = \beta_{51} = 0,\ \beta_{61} = \delta_{62} = 0,\ \delta_{61} = \alpha_{11},$$
$$\delta_{63} = 0, \beta_{72} = \delta_{71} = \delta_{73} = 0.$$
$$\delta_{31} = 0,\ \beta_{41} = \beta_{33},\ \beta_{32} = \beta_{43},\ \delta_{42} = 0,\ \beta_{31} = \frac{1}{2}(\alpha_{11} + \beta_{42}),$$
$$\beta_{42} = a_{11}, \beta_{83} = \delta_{81} = \delta_{82} = 0, \beta_{32} = \frac{1}{2}\beta_{43},\ \beta_{33} = \frac{1}{2}\beta_{41},$$
$$\beta_{33} = \beta_{41} = 0 = \beta_{32} = \beta_{43}, \delta_{53} = 0,\ \beta_{52} = \beta_{51} = 0, \beta_{53} = \alpha_{11}.$$

Taking into account these relations, we write the following:

$$\varphi(e_1) = \alpha_{11}e_1, \qquad \varphi(e_2) = \alpha_{11}e_2, \qquad \varphi(u_1) = \alpha_{11}u_1,$$
$$\varphi(u_2) = \alpha_{11}u_2, \qquad \varphi(u_3) = \alpha_{11}u_3, \qquad \varphi(v_1) = \alpha_{11}v_1,$$
$$\varphi(v_2) = \alpha_{11}v_2, \qquad \varphi(v_3) = \alpha_{11}v_3.$$

We complete the proof of theorem.

In this work, we present the following result obtained for local and 2-local $\frac{1}{2}$-derivations of the octonion and Okubo algebras.

Theorem[18]. Let be $\mathcal{L}$ an algebra, whose all $\frac{1}{2}$-derivations are trivial. Then any local and 2-local $\frac{1}{2}$-derivation of $\mathcal{L}$ is a trivial $\frac{1}{2}$-derivation.

From the theorem [18], we have following corollary:

Corollary. Let $\mathcal{O}$ Octonion and $\mathbb{O}$ are Okubo algebras over a field of characteristic zero. Then any local and 2-local $\frac{1}{2}$-derivation of the Octonion and Okubo a is a trivial $\frac{1}{2}$-derivation.

III. CONCLUSION

In this article, it is shown that the local and 2-local $\frac{1}{2}$-derivation of Octonion and Okubo algebras are trivial $\frac{1}{2}$-derivations, and general forms of $\frac{1}{2}$-derivations Octonion and Okubo algebras are presented.

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