Delay Risk Assessment for Job Shop Scheduling Considering Uncertain Processing Times

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Abstract. This study expresses the uncertainty of manufacturing processing time as a random variable and proposes a new production schedule evaluation method based on the probability distribution. We obtained results by using this proposed method. The results were compared with simulation results in order to clarify the effectiveness of the proposed method.

Keywords: Job shop, Scheduling, Processing time, Uncertainty.

1 Introduction

In many scheduling problems, the processing time of job is assumed to be a fixed value. However, in the case of make-to-order production such as molds and dies production, a machining tool and manufacturing processes to be assigned to the products are not determined when the orders are received, and it is difficult to estimate the processing times accurately. Under such a situation, in order to make a robust production schedule, a large margin of time may be added to the standard processing time for safety reasons. However, the extra estimated processing time leads to the decrease of productivity. Therefore, it is considered necessary to establish a production scheduling method that takes into account the uncertainty of processing time. This study proposes an evaluation method for production schedules that include manufacturing processes with uncertain processing times, and thus provides risk assessment criteria for uncertainty.

2 Literature review

Several studies on scheduling considering uncertain processing times have been proposed so far. For example, Sotskov et al. [1] investigated the effect of changes in processing time on the current schedule. Morita et al. [2] proposed a scheduling method that was robust to change in the degree of uncertainty by expressing the uncertain processing time distribution as a normal distribution in the scheduling problem of flexible flow shops. Ishibuchi et al. [3] proposed that the lower and upper limits of the processing time of each operation. They formulated the total required interval time for a flow shop scheduling problem. Morikawa et al. [4] attempted to obtain a schedule that minimized the make-span and was resistant to fluctuations in processing time. They introduced the concept of instability as a new evaluation criterion and measured the impact of a one-hour processing delay. This research shows that schedules with minimal instability are robust to variations in processing time when short delays occur in a small number of operations. Suwa et al. [5] proposed a scheduling approach that considered processing time delays when creating a schedule and sets a buffer as a time margin. This method uses a mathematical model to calculate the optimal amounts of buffers, divides the buffers into appropriate sizes, and places them on the critical path. However, it is difficult to introduce buffers. Although the processing delay is alleviated, there is a problem in which the make-span increases more than necessary time. Kouvelis et al. [6] targeted a two-machine flow shop scheduling problem with uncertain processing times.

3 How to evaluate a schedule including uncertain operations

3.1 Definition of symbols

(1) Job: J_i $(i = 1,2,\dots,m)$
A job represents an object to be processed, such as a part or product.

(2) Resource: R_j $(j = 1,2,\dots,n)$
A resource means a manufacturing resource such as a worker and a machining tool that manufactures products. A resource can only perform one operation at a time.

(3) Operation: $o_{i,j}^{l,p}$
An operation represents a process performed on a job by a resource, such as milling and drilling. Once an operation has started, it cannot be interrupted. Operation $o_{i,j}^{l,p}$ represents the *l*-th process in the process sequence of job J_i, and the *p*-th operation to be performed on resource R_j. The operation $o_{i,j}^{l,p}$ has the information about the starting time $st_{i,j}^{l,p}$, the finishing time $ft_{i,j}^{l,p}$, and processing time $pt_{i,j}^{l,p}$.

3.2 Delay risk assessment considering uncertain processing times

When a production schedule includes operations with uncertain processing times, it is impossible to reliably determine the optimal schedule. This is because the starting and finishing times of subsequent operations cannot be estimated accurately. Therefore, we propose an evaluation method that probabilistically estimates the starting and finishing times of production schedules with uncertain processing times of operations. This method re-expresses the obtained production schedule using the minimum processing times, and calculates the amount of change in the starting time and finishing time of each subsequent operation based on a probability calculation method.

In this study, we assume that the uncertain processing time t follows the probability density function $f(x)$, and conduct verification through experiments. The proposed method assumes that the probability density function of the uncertain processing time t is known, and the minimum value of the uncertain processing time t is also known.

Formulate the finishing time $ft(t_1, t_2, \dots, t_k)$ of an operation subject to an uncertain processing time $t_i (i = 1,2, \dots k)$. Let $w_i (i = 1,2, \dots, k)$ be the operation with uncertain processing time $t_i (i = 1,2, \dots k)$, and let $min_i \leq t_i \leq max_i$ for the uncertain processing time t_i. The uncertain processing time t_i is assumed to follow the probability density function $f_i(t_i)$. Let the finishing time of an operation be a function $ft(t_1, t_2, \dots, t_k)$ following the uncertain processing time $t_1, t_2, \dots, t_k$, as shown in Eq. (1).

Equation (1)

$$\begin{aligned} & \boldsymbol{ft(t_1, t_2, \dots, t_n)} \\ & \boldsymbol{= ft_x + max[f(1,2,n), f(1,3,n), \dots, f(1,n,n), f(2,3,n), f(2,4,n), \dots, f(2,n,n), \dots, f(n-1,n,n), f(n,n,n)]} \end{aligned}$$

However, $l, m, n \geq 1, l, m, n \in \mathbb{N}$, and $f(l, m, n)$ is shown in Eq. (2).

Equation (2)

$$\boldsymbol{f(l,m,n) = \begin{cases} t_l + max \left\{ \begin{array}{c} t_m - d_{ml} + max\big(t_{m+1} - d_{(m+1)m}, t_{m+2} - d_{(m+2)m}, \dots, t_n - d_{nm}, 0\big), t_m - d_{ml} + \\ t_{m+1} - d_{(m+1)m} + max\big(t_{m+2} - d_{(m+2)(m+1)}, t_{m+3} - d_{(m+3)(m+1)}, \dots, t_n - d_{n(m+1)}, 0\big), \\ \dots, t_m - d_{ml} + t_{m+1} - d_{(m+1)m} + \dots + max\big(t_{n-1} - d_{(n-1)n}, 0\big) \end{array} \right\} - d_{lx} \\ \quad (when\ \ l < m < n) \\ t_l + max(t_n - d_{nl}, 0) - d_{lx}\ (When\ \ m = n) \\ max(t_n - d_{nx}, 0)\ (When\ \ l = m = n) \end{cases}}$$

However, when the uncertain processing time $t_i (i = 1,2, \dots k)$ is larger than inter-operation delay parameter $d_{ij} (i, j = 1,2, \dots k)$, the finishing time of operation w_j is delayed. Inter-operation delay parameter d_{ij} can be divided into the following cases.

$$\begin{cases} \text{When operation } w_i \text{ affects operation } w_j\text{:} d_{ij} < \infty \\ \text{When operation } w_i \text{ does not affects operation } w_j\text{:} d_{ij} = \infty \\ d_{ij} = 0 (i = j) \end{cases}$$

Assuming that the uncertain processing times $t_1, t_2, \dots, t_k$ are independent, the joint probability density function $p(t_1, t_2, \dots, t_k)$ of $t_1, t_2, \dots, t_k$ can be found using the following formula.

$$p(t_1, t_2, \dots, t_k) = f_1(t_1) f_2(t_2) \dots f_k(t_k) \tag{3}$$

If the number of operations with uncertain processing time is k, the expected value E[X] of the finishing time of a certain operation is as follows.

$$\mathrm{E}[X] = \int \int \dots \int_D ft(t_1, t_2, \dots, t_k) p(t_1, t_2, \dots, t_k) dt_1 dt_2 \dots dt_k \tag{4}$$

$$D\text{:} min_1 \leq t_1 \leq max_1, min_2 \leq t_2 \leq max_2, \dots, min_k \leq t_k \leq max_k \tag{5}$$

4 Numerical experiments

4.1 Experimental conditions

In this study, the effectiveness of the proposed evaluation method was validated through computer experiments using the object-oriented language Smalltalk. The experiments were conducted on a Windows computer with a 12th Gen Intel(R) Core(TM) i7-1255U 2.60 GHz CPU and 16.00GB of memory.

4.2 Experimental results

In the case where there are two operations with uncertain processing times ($k = 2$), we calculated the probability distribution of finishing times using the proposed method, and also obtained data of the finishing times from 10,000 computer simulations. The results were compared by using histograms.

The production schedule used in the experiment was a job shop type production schedule with 10 jobs and 10 resources. Let the operation with uncertain processing time $t_1$be $o_{8,8}^{6,8}$, the operation with uncertain processing time t_2 be $o_{2,3}^{5,3}$, and the target operation be $o_{1,1}^{9,10}$. Figure1 shows a Gantt chart of the production schedule, when the processing time is set to the minimum value ($t_1 = 69$, $t_2 = 94$). Figure 1 also shows the relationship between the uncertain processing times t_1, t_2 and the target operation $o_{1,1}^{9,10}$.

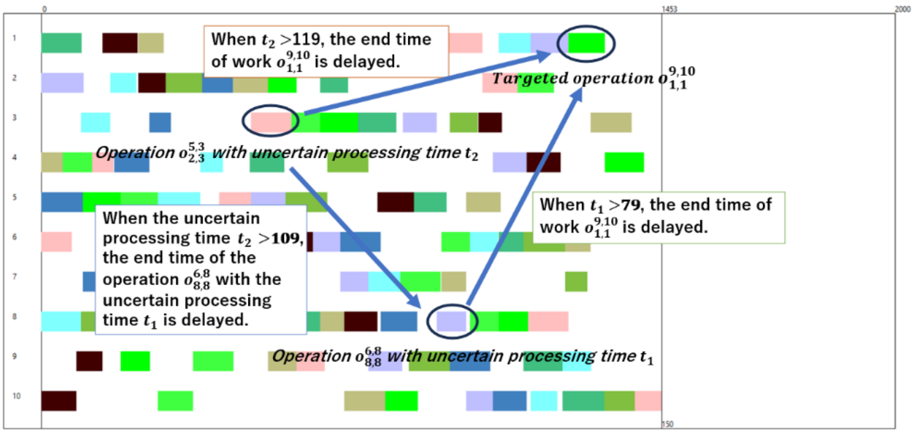

Fig. 1. Production schedule used in the experiment. The horizontal axis represents time, the vertical axis represents resources, and each rectangle represents an operation.

According to Kotani et al. [7], the Erlang distribution is well suited to the distribution of processing times that vary with probability. Therefore, the Erlang distribution is used in the computer experiments of this research. The probability density function $f_1(t_1)$ is shown in Eq. (6), and the probability density function $f_2(t_2)$ of the uncertain processing time t_2 is shown in Eq. (7).

$$f_1(t_1) = \frac{1}{256}(t_1 - 69)e^{\frac{-1}{16}(t_1 - 69)} \tag{6}$$

$$f_2(t_2) = \frac{1}{1024}(t_2 - 94)e^{\frac{-1}{32}(t_2 - 94)} \tag{7}$$

Let $f(t_1, t_2)$ be the function of the finishing time of operation $o_{1,1}^{9,10}$ with uncertain processing times t_1, t_2, and from $ft_x = 1321$, it is expressed by Eq. (8).

$$ft(t_1, t_2) = 1321 + (t_1 - 79, t_2 - 119, t_1 + t_2 - 188, 0) \tag{8}$$

Let $p\{ft(t_1, t_2)\}$ be the probability distribution of the function $ft(t_1, t_2)$ of the finishing time of operation $o_{1,1}^{9,10}$ with uncertain processing times t_1, t_2, then $p\{ft(t_1, t_2)\}$ is expressed by Eq. (9). However, when $ft(t_1, t_2) = 1321$, the probability distribution $p\{ft(t_1, t_2)\}$ becomes a probability function, and when $ft(t_1, t_2) > 1321$, the probability distribution $p\{ft(t_1, t_2)\}$ becomes a probability density function.

Equation (9)

$$p\{ft(t_1, t_2)\} = \begin{cases} \int_{94}^{188-x} \int_{69}^{79} f_1(t_1) \cdot f_2(t_2) dt_1 dt_2 \quad \{ft(t_1, t_2) = 1321\} \\ 0.0809 \times f_1\{ft(t_1, t_2) - 1242\} + h\{ft(t_1, t_2) - 1133\} \quad \{ft(t_1, t_2) > 1321\} \end{cases}$$

Figure 2 shows the results of comparing the theoretical value of the probability distribution obtained by Eq. (9) with the frequency of occurrence of the observed value based on 10,000 simulations.

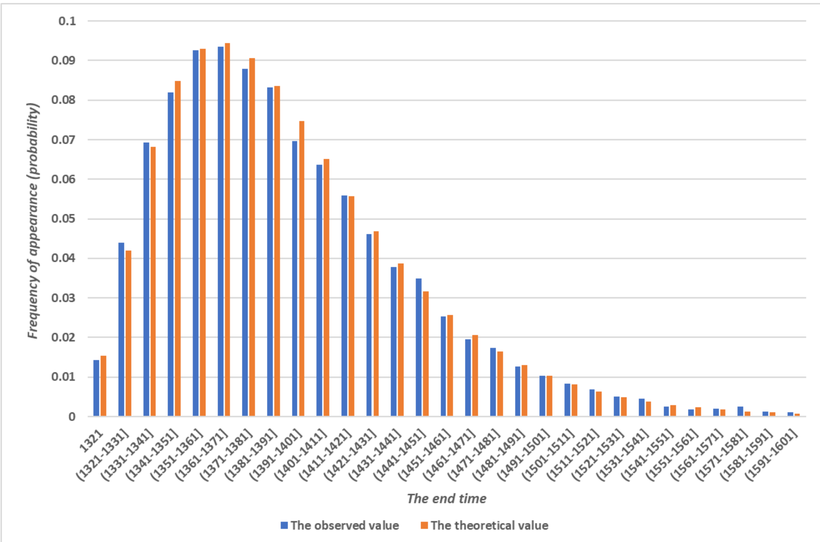

Fig.2. Comparison of the frequency of occurrence of the observed value based on 10,000 simulations and the theoretical value of the probability distribution obtained by Eq. (9).

The agreement between the probability distribution results derived by the proposed method and the histogram results obtained by the computer simulation was evaluated by applying the chi-square test. The chi-square test showed that the p-value was 1.0. This result means that both distributions show statistically significant agreement.

5 Conclusions

In this study, we expressed the uncertainty of processing time as a random variable and proposed a new evaluation method for production schedules based on probability distribution. We obtained probability distribution results by using this proposed method, and compared them with computer simulation results in order to verify the effectiveness of the proposed method.

In this study, we developed a schedule evaluation method that more accurately reflects the actual production environment by incorporating uncertainty into the schedule model. This allows you to evaluate job shop schedules that involve multiple operations with uncertain processing times. This can be expected to improve the accuracy and efficiency of production management.

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