A Study on Correction Method for Vertical Direction of Earthenware Fragments Based on 3D Measured Point Clouds

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Abstract

Restoration of earthenware fragments is one of the important roles in archaeology area. If earthenware fragments are excavated from ruins, the fragments are restored by manual operations. When earthenware are restored with computer, the vertical direction of fragments is required. In our previous method, a set of center points derived from cross-sections of fragments is estimated to determine the vertical direction. However, the vertical direction may be inverted because of the displacement of center points. Therefore, to manage the vertical inversion, this paper proposes a method to accurately correct the vertical direction of earthenware fragments based on 3D measured point clouds using linear regression.

Keywords: Earthenware fragments, Vertical direction correction, 3D measured point cloud, Linear regression, Center point displacement

1. Introduction

Relic earthenware is usually excavated from archeological sites in broken fragments, called earthenware fragments. The earthenware fragments are manually assembled to recreate the original shape. However, manual operations of earthenware restoration involve the risk of defacement and damage. If excavated earthenware fragments are found to have highly historical or cultural values, they are sometimes preserved without being restored in order to avoid such defacement or damage. However, the restoration of earthenware may be necessary for hypothesis formation through visualization or for the conservation and public presentation of cultural heritages. In recent years, it is expected to develop techniques to support the assembly of earthenware fragments using computers.

One of the studies on earthenware restoration is the study of estimating the vertical direction of an earthenware fragment using 3D measurement point cloud[1]. The shape restored in [1] is defined as a rotational shape, which is not an exact rotational surface of a vessel, presented [7]. Specifically, we target earthenware that is close to a cylinder with an acute angle at the intersection between the extended earthenware's generating line and the extended center axis of the earthenware's body. In [1], the cross-section of the rotational shape is approximated by circles. If the earthenware fragment is in the correct vertical direction, the center point of the circle approximating each cross-section of the earthenware fragment is aligned on the center axis of the earthenware. Therefore, the correct direction is estimated to be the one in which the average distance between the center point of the circle approximated by each cross-section and the projected point of the center point onto the central axis is the smallest. However, the average distance is the same for the correct vertical direction or the direction inversed 180° from the correct inversed direction.

This paper proposes a method for estimating the correct vertical direction of an earthenware fragment from a set of 3D measured points of the earthenware fragment for restoration assistance.

2. Related Works

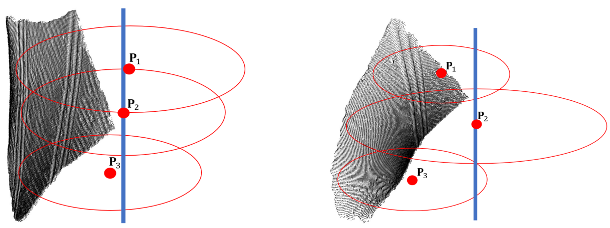
One of the related studies on earthenware restoration proposed a method for estimating the vertical direction and height position of earthenware fragments based on 3D measured point clouds[1]. In [1], the vertical direction of an earthenware fragment is estimated for an earthenware defined as a rotational shape. Specifically, earthenware with a rotational shape has circular cross-sections when being cut orthogonally to the central axis. If the earthenware fragment is in the correct vertical direction, the center points of the circles representing cross-sections lie on the central axis of the rotational shape. Let $\mathbf{P}_i(i = 0, ..., n)$ be the center point of the circle approximating each cross-section and $\mathbf{Q}_i(i = 0, ..., n)$ be the projected point of $\mathbf{P}_i$ onto the central axis C. the distance d_i between $\mathbf{P}_i$ and $\mathbf{Q}_i$ is obtained by Equation (1). If the earthenware fragment is in the correct vertical direction, d_i is small.

$$d_i = |\mathbf{P}_i - \mathbf{Q}_i|, (i = 0, ..., n) \tag{1}$$

Figure 1 shows circles estimated from the cross-sections of earthenware fragments, the center point $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ of the earthenware fragments, and the central axis C of the earthenware. As shown in Figure 1 (a), the center points estimated from the earthenware fragment indicating the correct vertical direction are aligned in a straight line. In contrast, if the earthenware is in an incorrect vertical direction, d_i between center point $\mathbf{P}_i$ of the circle approximating an earthenware fragment and $\mathbf{Q}_i$, which is the projected point of $\mathbf{P}_i$ onto central axis C, is larger. The center point $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$ on Figure 1 are not aligned in a straight line. Therefore, in [1], the posture with the smallest D_j expressed in Equation (2) is the correct vertical direction. the angle at which the earthenware fragment laid on the table and rotated is supposed to be j.

$$D_j = \sum_{i=0}^{n} d_i/(n+1), (j = 0, ..., 359) \tag{2}$$

However, the correct vertical direction of the estimated earthenware fragments cannot be determined because the average distance between the correct vertical direction and the 180° reversed vertical direction is $D_j = D_{j+180}$.



(a) Correct (b) Incorrect

Figure 1: center point of a circle approximated from the correct and incorrect vertical directions

In [2], a method was proposed for partial matching between the surface point cloud obtained by measuring the joining material and the point cloud representing the peeled surface of the stone tools. The method in [2] allows the spatial arrangement of the stone tools comprising the junction material to be recovered from the surface point cloud of the junction material and the measured point cloud of the stone tools. However, assembling the earthenware fragments requires estimating the coreect vertical direction, Therefore, we propose a method of estimating the correct vertical direction of a fragment of earthenware.

3. Proposed Method

3.1. summary

In this study, we propose a method for correcting the vertical direction of the earthenware fragments using a set of 3D measured points. The problem with previous work is that the distance d between center points $\mathbf{P}_i$ of the circle approximating an earthenware fragment and point $\mathbf{Q}_i$, which is the projection of $\mathbf{P}_i$ onto central axis C, is used to estimate the vertical direction. With this estimation, the correct vertical direction is ambiguous since the average distance is d_i in the correct vertical direction or in the vertical direction reversed by 180° as the posture of the fragments.

To resolve the ambiguity of the vertical direction, the correct vertical direction was estimated using the radii of the circles approximated from each section of the posture obtained in [1] and the heights of the earthenware fragment. Specifically, regression analysis is carried out using the heights of the earthenware fragment as the explanatory variable and the radius obtained from each cross-section as the explained variable. For example, if the radii increase with the heights of the earthenware fragments, the vertical direction obtained in [1] can be assumed to be correct. In contrast, if the radii become smaller as the heights of the earthenware fragment increase, the vertical direction obtained in [1], reversed by 180°, can be assumed to be the correct direction.

3.2. Extraction of radius distribution of earthenware fragments

The height of an earthenware in the correct vertical direction or in the vertical direction reversed by 180° from the correct vertical direction, as determined from [1], and the radii of the circles approximated from the cross-sections at several heights are extracted. In Figure 2, the vertical arrow on the earthenware fragment indicates the height and the horizontal arrows indicate the radii at each height.

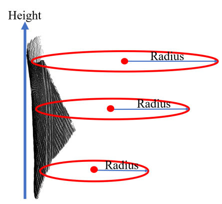
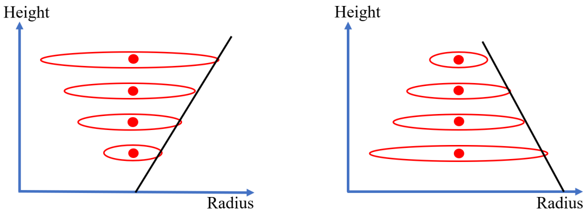


Figure 2: Radii and heights extraction

3.3. Vertical direction estimation by regression line

Utilize the radial distribution extracted in Section 3.2 for vertical direction estimation. Linear regression is performed with the height of the radial distribution as the explanatory variable and the radius as the explained variable. The correct vertical direction is estimated by comparing the slope of the obtained regression equation with the vertical direction obtained from[1]. In 3, the height of an earthenware fragment is represented on the vertical axis and the circle radius on the horizontal one, and the regression line obtained from the radius distribution is superimposed. For example, if the posture of an earthenware fragment estimated by [1] is the one shown in Figure 4 and the radius increases as the height increases as shown by Figure 3 (a), then the vertical direction of Figure 4 will be correct. In contrast, if the radius becomes smaller as the height increases as shown by Figure 3 (b), the opposite vertical direction of Figure 4 will be correct.



(a) Plus (b) Minus

Figure 3: The slope of the regression equation

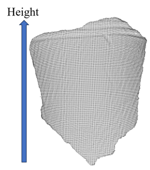


Figure 4: Vertical direction of earthenware fragment

3.4. Existence of outliers and countermeasures

Taubin's circle approximation method[3] is used to approximate circles in [1]. Figure 5 shows distance L between the end points of a cross-section. When distance L is short, the radius estimated by Taubin's circle approximation method is smaller than the original radius of the earthenware. Depending on the shape of the earthenware fragment, the accuracy of the radii of the approximated circles will be reduced. To reduce the impact of outliers due to reduced radius accuracy, robust regression methods that are robust to outliers based on robust statistics[4] are used. The Bisquare type of M-estimator is used as a robust regression method. In M-estimator, regression parameters are obtained by solving the estimating equation (3). Equation (4) is used in equation (3) as the function ψ.

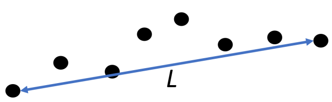


Figure 5: Distance L between end points of cross section

$$\sum_{i=1}^{n} \psi(y_i - \beta^T x_i)x_i = 0 \tag{3}$$

n is the total number of data, x_i and y_i are the data values, i is te data number, and β is the slope of the regression equation.

$$\psi(x'_i) = \begin{cases} (x'_i)\{1-(x'_i/c)^2\}^2 & |x'_i| \leq 0 \\ 0 & |x'| > 0 \end{cases} \tag{4}$$

$x'_i = x_i - \mu$, μ is the mean, and c is the threshold, and in this method, $c = 4.685$ according to [5].

4. Experimental Results

In this chapter, the effectiveness of the proposed method is verified. The CPU used for the experiment is Intel Core i7-1255U with 16.0GB RAM. Thirty-four fragments of earthenware stored at the Morioka City Museum of Archaeological Sites[6] were used in the experiment.

First, the vertical direction of the earthenware fragments is estimated using the method described in [1]. Table 1 shows the total number of fragments, the number of fragments for which the correct vertical direction is estimated, the number of fragments for which the vertical direction is estimated by inverting the correct vertical direction by 180°, and Number of earthenware fragments that failed to be estimated. The experiment for this method targets 21 earthenware fragments inverted 180° from the correct vertical direction using the method described in [1].

Table 1: Number and percentage of inverted earthenware fragments

total	correct direction	inversed direction	failure
34	9	21	4
100%	26%	61%	11%

Next, we conducted an experiment using this method on 21 earthenware fragments for which the vertical direction was obtained by reversing the correct vertical direction by 180° using [1]. The least-squares method is also used to estimate the vertical direction as a comparison to the bisquare type of M-estimator. Table 2 shows the number of earthenware fragments for which the vertical direction was estimated to be inverted 180° from the correct vertical direction using [1] and the number of earthenware fragments for which the vertical direction was estimated to be inverted 180° from the correct vertical direction using the least squares method and the bisquare type of M-estimator. As a result of the experiment, the number of earthenware fragments that are inverted 180° from the correct vertical direction in [1] could be reduced from 21 to 4. Therefore, the percentage of inverted fragments in [1] is 70%, whereas the number of inverted fragments is reduced to about 13% by the present method. However, earthenware numbers 19, 20, 24, and 34 are incorrect. For example, earthenware fragment number 19 is located near the middle of the earthenware, and its slope is ambiguous. As shown in Figure 6, the radius of the circle near the center where fragment number 19 is located does not change with increasing or decreasing height. Therefore, the method needs to be improved to increase precision.

Table 2: Number and percentage of inverted earthenware fragments

number	[1]	least-squares method	M-estimator Bisquare type
2	○	○	
3	○		
4	○		
5	○		
7	○		
9	○		
12	○		
13	○		
14	○		
16	○		
17	○		
18	○		
19	○	○	○
20	○	○	○
21	○		
23	○		
24	○	○	○
25	○		
27	○	○	
28	○	○	
34	○	○	○
total	21	7	4
percentage	70%	23%	13%

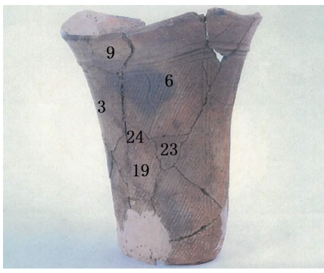


Figure 6: assembled earthenware

5. Conclusion and Future Works

In the study, we examined a method to estimate the vertical direction by linear regression of the radius distribution of the earthenware fragments and using the slope of the regression. In [1], the percentage was 70%, whereas, in the proposed method, it was about 13%. However, there are earthenware fragments that are in the correct vertical direction in [1], but are inverted in the proposed method. Therefore, the method needs to be improved to increase accuracy.

Acknowledgments

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