Forced Vibration Analysis Nanoplates Resting on Elastic Foundations Taking into Account Flexoelectric Effect

Huyen Truong Thi Huong, Minh Phung Van, Ke Tran Van and Thom Do Van

November 28, 2022

Forced vibration analysis nanoplates resting on elastic foundations taking into account flexoelectric effect

Truong Thi Huong Huyen, Phung Van Minh*, Tran Van Ke, and Do Van Thom*
Faculty of Mechanical, Le Quy Don Technical University
*Email: minhpv.mta@gmail.com; thom.dovan.mta@gmail.com

Abstract. This work uses the finite element method for the first time to simulate the dynamic response of nanoplates resting on a two-parameter elastic foundation under the flexoelectric effect. The calculation formulas are derived from the new type of the shear deformation theory, which is a simple shear deformation theory. In which, the displacement component in the direction perpendicular to the plane of the plate is subdivided into the bending strain component and shear strain component. Therefore, this approach also does not need any shear correction factor. The finite element equations are developed on a four-node quadrilateral, where each node has six degrees of freedom. The comparison with published results shows the reliability of this theory. The numerical data clearly shows the influence of the flexoelectric effect, elastic foundation, and thickness on the dynamic response of nanoplates. These results can be referenced in the design, fabrication, and use of nanostructures in practice.

Keywords: *nanoplates, elastic foundation, flexoelectric, forced vibration, FEM.*

1. Introduction

Piezoelectric nanostructures are employed in sensors, actuators, energy harvesters, and more as science and technology advance. Flexoelectricity-especially strain gradient-induced electric polarization-is ubiquitous in these formations. Several investigations have shown that nanoplate mechanical behavior accounts for the flexoelectric effect. Yan [1] studied static bending and free vibration of piezoelectric nanoplates using classical plate theory and flexoelectricity. Yang et al. [2] used Kirchoff's plate theory (classical plate theory-CPT) to provide explicit solutions for nanoplates' static bending and free vibration response. The theory incorporated piezoelectric and flexoelectricity. Li et al. [3] analyzed static bending and free vibration of the circular microplate. The equations were formed from the CPT, the solution was calculated analytically, and the findings showed the influence of flexoelectric effect on mechanical response.

There has also been research done on the buckling, vibration, and static bending responses of nanostructures while taking into consideration the flexoelectricity effect [4-10]. In which the aforementioned studies are founded on the traditional plate theory as well as the first-order shear deformation theory.

As can be observed from the works that have come before, there has not been any research done to examine the dynamics of nanoplates resting on an elastic foundation that includes the flexoelectric effect. This body of work is a scientific effort that contributes to the design, production, and practical use of nanostructures.

2. Forced oscillation equation of the nanoplate taking into account the flexoelectric effect

This work focuses on the forced oscillation of nanoplates modeled as shown in Figure 1. The plate has geometric parameters including length a, width b, and thickness h. The whole mechanical system is supported on an elastic foundation with two coefficients k_w and k_s.

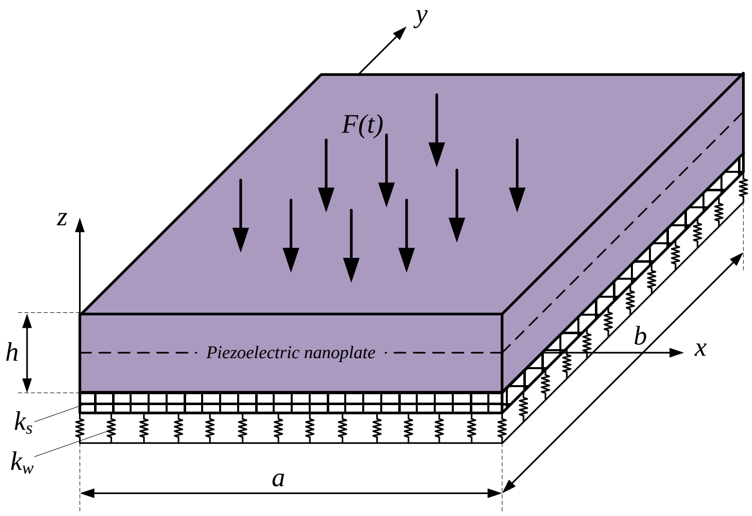

Figure 1. The model of nanoplate resting on a two-parameter elastic foundation subjected to dynamic loads

In order to study the mechanical response of plate structures, there are different sheet theories to apply. This work uses the theory of shear deformation hyperbolic sine functions [11], therefore, the displacement field at any point of the plate is written as follows:

$$u_x = -zu_{b,x} - f(z)u_{s,x};\ u_y = -zu_{b,y} - f(z)u_{s,y};\ u_z = u_b + u_s \tag{1}$$

in which $l_z = z - \vartheta_z$, $\vartheta_z = h.\sin\frac{z}{h} - z.\cosh\frac{1}{2}$; u_x, v_y, and u_z are the displacements in the *x*-, *y*-, and *z*-directions at one point within the plate. The comma is the derivative of the variable immediately following it. Moreover, this study disregards the size effect. To account for the impact of the flexoelectricity effect on the nanoplates, this study employs strain gradient theory, as demonstrated in Equation (3) below, which has already been confirmed in the literature [14].

Taking the displacement derivative with respect to the coordinates, one gets the deformation components:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -zu_{b,xx} - l_z u_{s,xx} \\ -zu_{b,xx} - l_z u_{s,yy} \\ -zu_{b,xy} - 2l_z u_{s,xy} \end{Bmatrix} = z\underbrace{\begin{Bmatrix} \varepsilon_{bx} \\ \varepsilon_{by} \\ \gamma_{bxy} \end{Bmatrix}}_{\varepsilon_b} + l_z \underbrace{\begin{Bmatrix} \varepsilon_{sx} \\ \varepsilon_{sy} \\ \gamma_{sxy} \end{Bmatrix}}_{\varepsilon_s};\ \boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \vartheta_{z,z} \begin{Bmatrix} u_{s,x} \\ u_{s,y} \end{Bmatrix} = \vartheta_{z,z}\boldsymbol{\gamma}_0 \tag{2}$$

This study assumes that the strain gradient in the thickness direction is much smaller than the strain gradients in the *x*- and *y*-directions, and that the strain gradient in the *z*-axis is neglected. The components of strain gradient include:

$$\boldsymbol{\eta} = \begin{Bmatrix} \eta_{xxz} = -u_{b,xx} - l_{z,z}u_{s,xx} \\ \eta_{yyz} = -u_{b,yy} - l_{z,z}u_{s,yy} \end{Bmatrix} = \begin{Bmatrix} -u_{b,xx} \\ -u_{b,yy} \end{Bmatrix} + l_{z,z} \begin{Bmatrix} -u_{s,xx} \\ -u_{s,yy} \end{Bmatrix} = \boldsymbol{\eta}_b + l_{z,z}\boldsymbol{\eta}_s \tag{3}$$

The nanoplate is divided into four-node elements, each node has six degrees of freedom:

$$\boldsymbol{q}_e = \sum_{i=1}^{4} \left\{ u_{bi}, u_{si}, u_{b,xi}, u_{s,xi}, u_{b,yi}, u_{s,yi} \right\}^T \tag{4}$$

So, one gets:

$$\{u_b, u_s\} = \sum_{i=1}^{4} \left\{ H_i \{u_{bi}, u_{si}\} + H_{i+1} \{u_{b,xi}, u_{s,xi}\} + H_{i+2} \{u_{b,yi}, u_{s,yi}\} \right\} = \{\boldsymbol{H}_b, \boldsymbol{H}_s\} \boldsymbol{q}_e \tag{5}$$

in which, H_j is the Hermit interpolation function.

The displacement vector at any point inside the element is then interpolated using the nodal displacement vector.

$$\boldsymbol{u} = \{u_b, u_s, u_{b,x}, u_{s,x}, u_{b,y}, u_{s,y}\}^T = \boldsymbol{H}.\boldsymbol{q}_e \tag{6}$$

According to this formulation, strain vectors are generated as follows using the nodal displacement vector:

$$\boldsymbol{\varepsilon}_b = \boldsymbol{B}_1\boldsymbol{q}_e;\ \boldsymbol{\varepsilon}_s = \boldsymbol{B}_2\boldsymbol{q}_e;\ \boldsymbol{\gamma}_0 = \boldsymbol{B}_3\boldsymbol{q}_e;\ \boldsymbol{\eta}_b = \boldsymbol{B}_4\boldsymbol{q}_e;\ \boldsymbol{\eta}_s = \boldsymbol{B}_5\boldsymbol{q}_e \tag{7}$$

When the flexoelectric effect is taken into consideration, the stress components and electric displacement vector for a nanoscale dielectric material are expressed as follows:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k;\ \chi_{ijm} = -f_{kijm}E_k;\ P_i = c_{ijk}\varepsilon_{jk} + \kappa_{ij}E_k + f_{ijkl}\eta_{jkl} \tag{8}$$

in which c_{ijkl}, e_{kij}, f_{kijm} and κ_{ij} are the components of elastic, piezoelectric, flexoelectric and permittivity constant tensor; they are the material parameters. T_{ij} is the stress tensor, which is similar to that of the traditional elastic foundation. P_i is the electric displacement vector, and χ_{ijm} is the moment stress tensor or the higher-order stress tensor.

From the equations of the strain components, we can derive the following formulae for the stress and electric displacement vector:

$$\boldsymbol{\sigma} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{11} & 0 \\ 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - e_{31} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} E_z = \boldsymbol{C}_b\boldsymbol{\varepsilon}\text{-}\tilde{\boldsymbol{E}}\ ;\ \boldsymbol{\tau} = \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} c_{66} & 0 \\ 0 & c_{66} \end{bmatrix} \boldsymbol{\gamma} = \boldsymbol{C}_s\boldsymbol{\gamma} \tag{9}$$

$$\boldsymbol{\chi} = \begin{Bmatrix} \chi_{xxz} \\ \chi_{yyz} \end{Bmatrix} = -f_{14} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} E_z\ ;\ P_z^0 = e_{31}\left(\varepsilon_{xx} + \varepsilon_{yy}\right) + \kappa_{33}E_z + f_{14}\left(\eta_{xxz} + \eta_{yyz}\right) \tag{10}$$

where $f_{14} = f_{3113}$ and $f_{14} = f_{3223}$ [12].

The electrical field is computed as follows using the partial derivative of electrical potential:

$$E_z = \frac{e_{31}}{\kappa_{33}}\left(u_{b,xx} + u_{b,yy}\right)z + \frac{e_{31}}{\kappa_{33}}\left(u_{s,xx} + u_{s,yy}\right)l_z + \frac{f_{14}}{\kappa_{33}}\left(u_{b,xx} + u_{b,yy}\right) + \frac{f_{14}}{\kappa_{33}}\left(u_{s,xx} + u_{s,yy}\right)l_{z,z} \tag{11}$$

The expression of the deformation potential energy of the plate element is shown as follows:

$$\Pi = \frac{1}{2}\int_V \left(\boldsymbol{\varepsilon}^T \boldsymbol{C}\boldsymbol{\varepsilon} + \boldsymbol{\gamma}^T \boldsymbol{\tau} - \boldsymbol{\varepsilon}^T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \frac{e_{31}^2}{k_{33}} \left(u_{b,xx} + u_{b,yy} \right).z \right) dV + \frac{1}{2}\int_S \left(k_w u_z^2 + k_s \left(u_{z,x}^2 + u_{z,y}^2 \right) \right) dS$$
$$= \frac{1}{2} \boldsymbol{q}_e^T \boldsymbol{K}_e \boldsymbol{q}_e \tag{12}$$

where k_w and k_s are the two coefficients of the elastic foundation, and $\boldsymbol{K}_e$ is the element stiffness matrix.

The expression for the kinetic energy of the plate element is shown as follows:

$$T = \frac{1}{2}\int_V \left(\rho \left\{ \dot{u}_x, \dot{v}_y, \dot{w}_z \right\} \left\{ \dot{u}_x, \dot{v}_y, \dot{w}_z \right\}^T \right) dV = \frac{1}{2} \dot{\boldsymbol{q}}_e^T \int_V \left(\rho \boldsymbol{H}^T \boldsymbol{G}^T \boldsymbol{G} \boldsymbol{H} \right) dV \dot{\boldsymbol{q}}_e \tag{13}$$

where ρ is the density of the material.

Following is the formula for the external force exerted on the plate:

$$W_e = \int_{\Omega_e} \left(w_b + w_s \right)^T \boldsymbol{P}_\Omega dS = \boldsymbol{q}_e^T \int_{S_e} \left(\boldsymbol{H}_b + \boldsymbol{H}_s \right)^T \boldsymbol{P}_\Omega dS = \boldsymbol{q}_e^T \boldsymbol{F}_e \tag{14}$$

After using Hamilton's principle, one gets the forced oscillation equation of the plate as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{F} \tag{15}$$

where $\boldsymbol{M} = \sum_e \boldsymbol{M}_e \ddot{\boldsymbol{q}}_e$; $\ddot{\boldsymbol{q}} = \sum_e \ddot{\boldsymbol{q}}_e$; $\boldsymbol{K} = \sum_e \left(\boldsymbol{K}_e + \boldsymbol{K}_e^f \right)$; $\boldsymbol{q} = \sum_e \boldsymbol{q}_e$; $\boldsymbol{F} = \sum_e \boldsymbol{F}_e$.

In case the internal frictional resistance of the structure is taken into account, the equation of forced vibration with resistance is in the form:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{F} \tag{16}$$

where $\boldsymbol{C} = \alpha \boldsymbol{M} + \beta \boldsymbol{K}$, α, and β are two coefficients calculated from the first two natural frequencies of the nanoplate.

For the problem of free oscillation without resistance, the equation has the following form:

$$\left\{ \boldsymbol{K} - \omega^2 \boldsymbol{M} \right\} \boldsymbol{q}_0 = \boldsymbol{0} \tag{17}$$

Solving equation (17), one gets the eigenfrequency and corresponding eigenforms. Solving Equation (16) will find the motion characteristics of the plate such as displacement, velocity, and acceleration of the plate's oscillation under the action of dynamic loads. For a plate subjected to the fully simply supported boundary condition (SSSS), the constraint condition is $w_b = 0$ and $w_s = 0$. To solve equation (16), this work uses the Newmark method of direct integration.

3. Verification study

In this section, a number of comparison issues will be performed to validate the suggested theory and mathematical model. In this study, the numerical findings of deflection and stress are compared to accurate published data.

Example 1: Consider an SSSS square plate with geometry parameters a/b=1, the plate thickness $h=a/10$, and $a/20$, material parameters E=380 Gpa, $\rho = 3800$ kg/m^3 and ν = 0.3. Two elastic foundation parameters are normalized as follows:

$$K_w^* = \frac{k_w a^4}{D_0};\ K_s^* = \frac{k_s a^2}{D_0};\ D_0 = \frac{E_0 h^3}{12\left(1-\nu^2\right)};\ E_0 = 70\text{ GPa} \quad (18)$$

The parameter to be compared is the first natural frequency, which is calculated by the dimensionless formula as follows.: $\varpi = \omega_1 h\sqrt{\rho_0 / E_0}$; $\rho_0 = 2707$ (kg/m^3).

The comparative non-dimensional fundamental frequencies of the plate generated by this work and the analytical solution [13] are shown in Table 1, where various mesh sizes are introduced by this study. It can be observed that the results converge with the 64-element mesh.

Table 1. The comparative nondimensional fundamental frequency ϖ of the plate supported by a two-parameter elastic base.

K_w^*	K_s^*	a/h	Analytical solution [13]	This work			
				16 elements	64 elements	100 elements	256 elements
100	0	10	0.1162	0.1129	0.1154	0.1157	0.1160
	100		0.1619	0.1591	0.1612	0.1614	0.1617
	0	20	0.0298	0.0289	0.0296	0.0296	0.0297
	100		0.0411	0.0404	0.0409	0.0410	0.0411

4. Numerical results

The nanoplate is subjected to the SSSS boundary; geometry parameters are h=20 nm, $a=b=50h$, and the material properties are c_{11}=102 GPa; c_{12}=31 GPa; c_{33}=35.50 GPa; e_{31}=-17.05 C/m^2; k_{33}=1.76.10^{-8} C/(Vm). The plate is rested on the two-parameter elastic foundation with with k_w and k_s.

The uniformly applied load on the plate has the expression:

$$P = P_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)F(t);\quad F(t) = \begin{cases} \sin\left(\pi\dfrac{t}{t_1}\right) & 0 \le t \le t_1 \\ 0 & t > t_1 \end{cases} \quad (19)$$

where, the load amplitude is P_0=5.10^4 N/m^2 , and $t=0.8t_1$.

Non-dimensional parameters are calculated as follows:

$$w^* = \frac{10^3 c_{11} h_0^3}{12 P_0 a^4} w;\ f_{14}^* = \frac{f_{14}}{f_{14}^0};\ k_1^* = \frac{k_w a^4}{D_f};\ k_2^* = \frac{k_s a^2}{D_f};\ D_f = \frac{c_{11} h_0^3}{12};\ \left\{\sigma_x^*, \tau_{xz}^*\right\} = \frac{1}{P_0}\left\{\sigma_x, \tau_{xz}\right\} \quad (20)$$

with f_{14}^0 = 10^{-7} C/m and $h_0 = a/50$.

4.1. Influence of the flexoelectric effect

Consider an SSSS nanoplate resting on a two-parameter foundation (K_w^*=100, K_s^*=10). To clearly see the influence of parameter f_{14} on the dynamic response of the plate, this coefficient is changed so that f_{14}^* varies from 0 to 5 (When f_{14}^*=0 corresponds to the case of ignoring the flexoelectric effect.). Numerical results of displacement response w^* and stress σ_x^* over time are shown in Figures 3 and 4, and the data show that:

- When taking into account the effect of flexoelectric effect (i.e. f_{14} is non-zero), the maximum displacement of the plate is reduced. When the external force stops acting on the plate, the plate oscillates gradually.

- Although the coefficient f_{14} increases, the maximum displacement of the plate decreases. However, the transformation law of the maximum shear stress τ_{xz}^* of the plate is different from the transformation law of displacement w and normal stress σ_x^*. This is because the parameter f_{14} affects the stiffness of the plate. This is a special phenomenon, which is quite different from conventional structures, regardless of the flexoelectric effect.

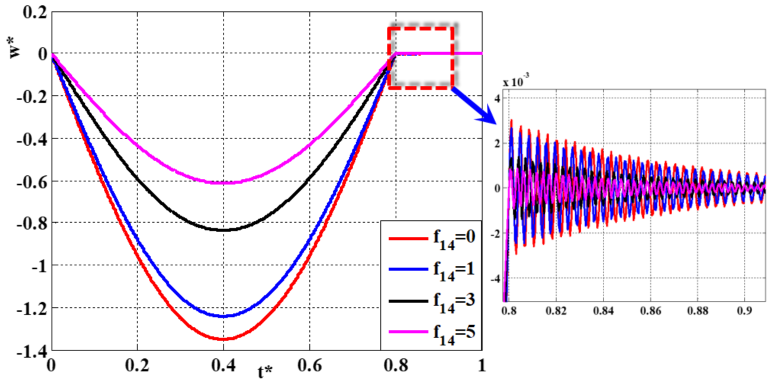

Figure 3. Variation of displacement w^* depends on time and f_{14}, $t^* = t/t_1$

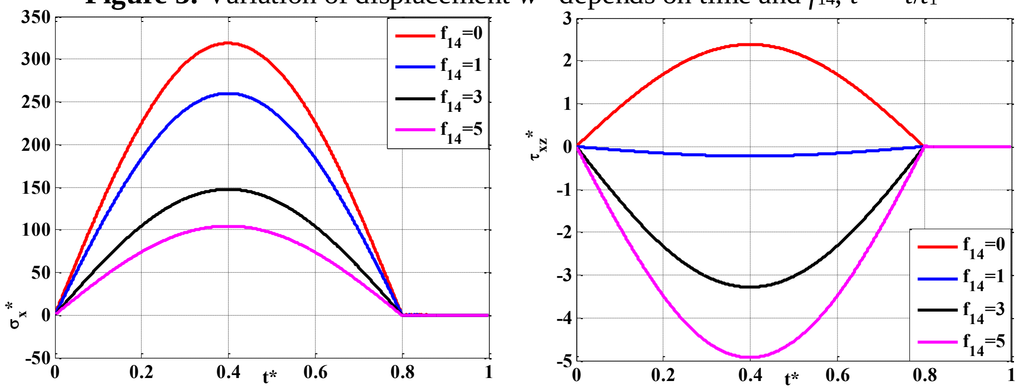

Figure 4. Variation of stresses σ_x^*, τ_{xz}^* depends on time and f_{14}

4.2. Influence of the plate thickness h

The plate thickness h is changed so that the ratio a/h varies from 50-100; the results of the dynamic response of the plate are shown in Figures 5 and 6. One can see that as the thickness of the plate is decreased, the maximum displacement and maximum normal stress of the plate increase. However, the increase in the maximum displacement is not uniform. As the plate thickness decreases, it is clear that when the plate thickness varies between $a/50$ and $a/80$, the maximum displacement and maximum stress change more when the plate thickness varies from $a/80$ to $a/150$.

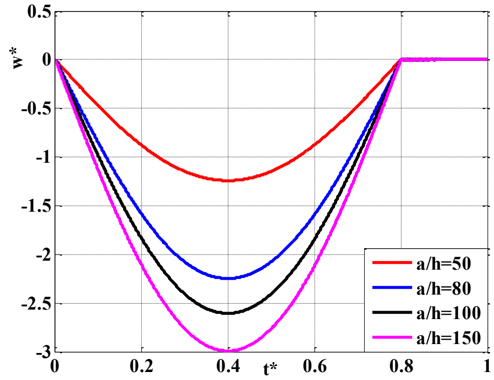

Figure 5. Variation of displacement *w** depends on time and ratio *a*/*h*

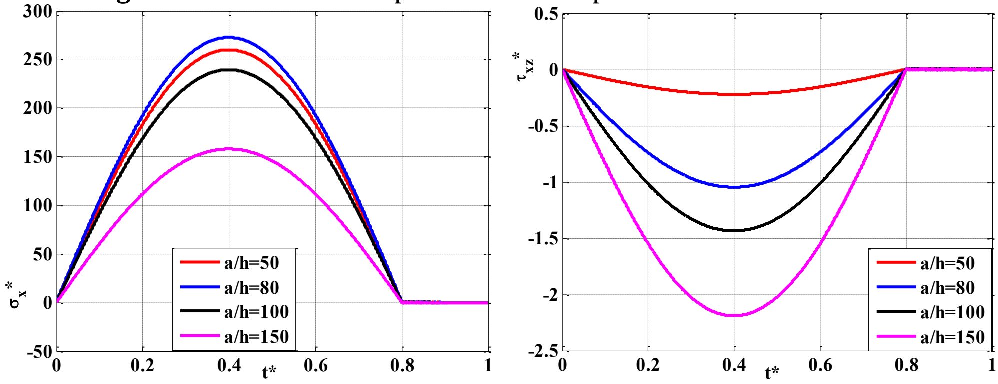

Figure 6. Variation of stresses σ_x^*, τ_{xz}^* depends on time and ratio *a*/*h*

4.3. Influence of elastic foundation

The elastic foundation coefficients are changed so that the first elastic foundation coefficient varies from 10 to 200, while the second one varies from 1 to 20. The numerical results of displacement response *w** and normal and shear stresses of nanoplatets are given as shown in Figures 7 and 8. The outcomes show that the higher the elastic foundation coefficient, the greater the energy of the plate, and the harder the plate becomes. Therefore, the maximum displacement and maximum stress of the nanoplate are also reduced.

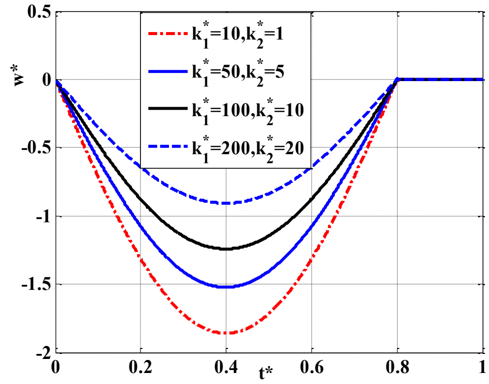

Figure 7. Variation of displacement *w** depends on time and elastic foundation coefficients

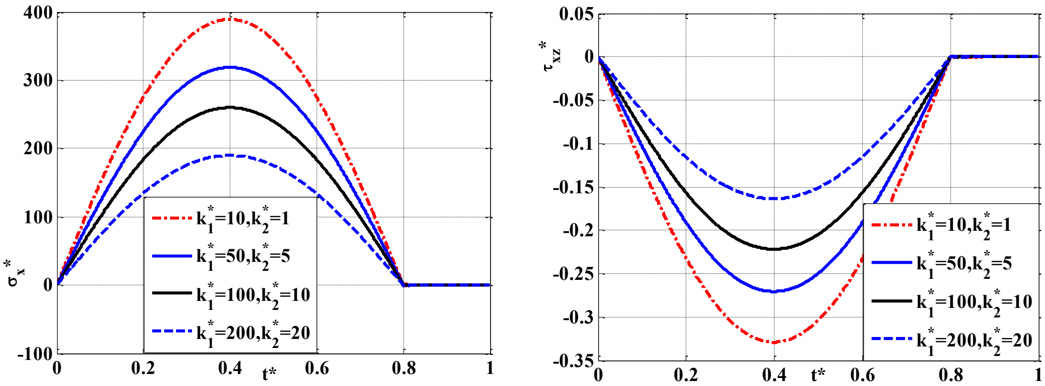

Figure 8. Variation of stresses σ_x^*, τ_{xz}^* depends on time and elastic foundation coefficients

5. Conclusions

This paper examines the dynamic response of nanoplates, including the flexoelectic effect, by combining the novel shear deformation theory with the finite element approach. This theory has numerous benefits, including simplicity, computation convenience, and no requirement for a shear correction factor, however it still describes exactly the mechanical reaction of the structure. The veracity of computational theory is determined by comparing it to published findings. This study also analyzes the effect of several material characteristics, geometric parameters, and elastic substrates on the dynamic response of nanoplates.

References

[1] Yan Z. Size-dependent bending and vibration behaviors of piezoelectric circular nanoplates. Smart Materials and Structures 2016, **25**, 035017, pp. 1-13.

[2] Yang W, Liang X, Shen S. Electromechanical responses of piezoelectric nanoplates with flexoelectricity. Acta Mechanica 2015, **226**, pp. 3097–3110.

[3] Li A, Zhou S, Qi L. Size-dependent electromechanical coupling behaviors of circular micro-plate due to flexoelectricity. Applied Physics A 2016, **122**, pp. 1-118.

[4] Wang X, Zhang R, Jiang L. A Study of the Flexoelectric Effect on the Electroelastic Fields of a Cantilevered Piezoelectric Nanoplate. International Journal of Applied Mechanics 2017, **9** (4), 1750056, pp. 1-25.

[5] He L, Lou J, Zhang A, Wu H, Du J, Wang J. On the coupling effects of piezoelectricity and flexoelectricity in piezoelectric nanostructures. AIP advanced 2017, **7**, 105106.

[6] Ebrahimil F, Barati MR. Static stability analysis of embedded flexoelectric nanoplates considering surface effects. Applied Physics A 2017, **123**, pp. 1-15.

[7] Amin G, Yaghoub TB, Hossein G. Nonlinear thermo-electromechanical vibration analysis of size-dependent functionally graded flexoelectric nano-plate exposed magnetic field. Archive of Applied Mechanics 2020, **90**, pp. 2025–2070.

[8] Amir S, Akbar-Zarei HB, Khorasani M. Flexoelectric vibration analysis of nanocomposite sandwich plates. Mechanics Based Design of Structures and Machines: An International Journal 2020, **48** (2), pp. 146-163.

[9] Ghobadi Amin, Beni YT, Golestanian H. Size dependent thermo-electro-mechanical nonlinear bending analysis of flexoelectric nano-plate in the presence of magnetic field. International Journal of Mechanical Sciences 2019, **152**, pp. 118–137.

[10] Ghobadi Amin, Beni YT, Golestanian H. Size Dependent Nonlinear Bending Analysis of a Flexoelectric Functionally Graded Nano-Plate Under Thermo-Electro-Mechanical Loads. Journal of Solid Mechanics 2020, **12** (1), pp. 33-56.

[11] Thai-Huu T, Dong-Ho C. Finite element formulation of various four unknown shear deformation theories for functionally graded plates. Finite Elements in Analysis and Design 2013, **75**, pp. 50-61.

[12] Shu LL, Wei XY, Pang T, Yao X, Wang CL. Symmetry of flexoelectric coefficients in crystalline medium. Journal of Applied Physics 2011, **110**, 104106.

[13] Baferani AH, Saidi AR, and Ehteshami H. Accurate solution for free vibration analysis of functionally graded thick rectangular plates resting on elastic foundation. Composite Structures 2011, **93**, 7, pp. 1842–1853.

[14] Doan HD, Do VT, Pham HC, Phung VM, Nguyen XN. Vibration and static buckling behavior of variable thickness flexoelectric nanoplates. Mechanics Based Design of Structures and Machines 2022, pp. 1-29.