Optimization Design of Coaxial Reverse Rotor Axis Based on Response Surface Method

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January 20, 2025

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Abstract. Aiming at the rotor shaft of the common axis reverse transmission system, the optimization design of the transmission system structure was studied. By response surface optimization method, a response surface optimization model based on the finite element simulation analysis software ANSYS workbench was established. According to the fitted response surface model, the structure optimization design of the inner and outer rotor shafts was carried out. Taking a set of inner and outer rotor shafts as an example, effectiveness of optimization of the rotor shaft structure of a common axis reverse transmission system based on response surface method is verified.

Keywords: Response Surface Method,Rotor Shaft,Optimization Design.

Compared with conventional single rotor helicopters, coaxial reversal twin rotor high-speed helicopters have two sets of rotor systems for coaxial reversal. That is, the engine power needs to simultaneously drive the upper and lower layers of rotors arranged on the same axis to rotate in the opposite direction at a constant speed to counteract the rotor reverse torque. Therefore, the structure of the coaxial reversal main reducer is more complex and the weight is much greater than that of the same tonnage single rotor helicopter main reducer. In the coaxial reverse high-speed helicopter transmission system, the weight of the rotor shaft accounts for a large proportion of the entire transmission system. Therefore, optimizing the design of the rotor shaft is one of the important measures to reduce weight in the coaxial reverse transmission system.

Structural optimization design has developed into an independent discipline since 1960[1]. Subsequently, researchers have adopted various methods such as linear programming, gradient projection, feasible direction method, and penalty function method to solve structural optimization problems with different constraints such as stress, displacement, and frequency. In 1995, Houten, Schoofs, Campen, and others[2] conducted a systematic study on the application of response surface methodology in structural optimization. In 2000, Haftka, Stander, Roux, and the Structural[3～5] and Multidisciplinary Optimization Research Group of the University of Florida conducted a comprehensive and systematic study on the application of response surface methodology in structural and multidisciplinary optimization. Currently[6～7], response surface optimization method has been widely used in chemical industry, biology, food science, engineering, and other fields[8].

This article establishes a rotor shaft structure optimization model based on the response surface method for the coaxial reverse transmission system. The optimization parameters and design variables of the rotor shaft structure are determined, and experimental design is carried out according to the central composite design method. The response surface model is fitted based on the calculation results and the accuracy of the model is verified. Finally, the optimized design of the rotor shaft is carried out through the fitted response surface model to achieve the goal of optimizing and reducing weight.

1 Basic Theory of Response Surface Methodology

Response Surface Method (RSM), also known as Response Surface Method, is an optimization statistical method that seeks optimal process parameters by analyzing response surfaces and contour lines, and uses multiple quadratic regression equations to fit the functional relationship between response values and factors[9]. It is a combination of statistics and mathematics, and its basic idea is to assume a mathematical expression (usually a low order polynomial) consisting of design variables and state variables with unknown coefficients. Based on the actual relationship between design variables and response variables, a response surface model is fitted and established. In most cases, it is necessary to use numerical theory and experimental analysis as a basis to perform sampling calculations within the domain of design variables to determine the relationship between design variables and response variables. The advantage of response surface methodology is that it can continuously analyze the various levels of response variables during the design variable optimization process, overcoming the shortcomings of orthogonal experiments that can only analyze isolated test points and cannot provide intuitive graphics.

2 Response surface methodology optimization process

The optimization process of response surface methodology mainly consists of three stages. The first stage is to determine independent design variables and their levels; The second stage is to select appropriate experimental design schemes and models for prediction and validation; Finally, obtain the response surface graph.

2.1 Approximate function

In general, the relationship between response variables and design variables is unknown, so the focus of response surface methodology is to determine an approximate functional model through parameter fitting. The form of functional models is one of the core components of response surface methodology. There are various forms of response surface function models, such as polynomial models, radial basis function models, and kriging models. Usually, low order polynomials such as first-order and second-order are often used in polynomial models. Compared with the first-order polynomial model, which is a linear function model, the second-order polynomial model is a nonlinear function model that can more accurately fit the nonlinearity of the function. The common formula for second-order polynomial response surface models is:

$$\hat{y} = \beta_0 + \sum_{j=1}^{n} \beta_j x_j + \sum_{j=1}^{n} \beta_{jj} x_j^2 + \sum_{i=1}^{n} \sum_{i \le j}^{n} \beta_{ij} x_i x_j$$

In the formula, n is the number of design variables; x_i and x_j are two different variables; β_0、β_j、β_{jj} and β_{ij} are undetermined coefficients, which are determined using the least squares method based on the correspondence between the obtained design variables and response variables.

2.2 Response surface meshing degree evaluation

In the practical application of response surface methodology, in order to determine the fitting degree of the response surface function model, it is necessary to evaluate its fitting accuracy. The following are common evaluation indicators.

(1) Multiple correlation coefficient R^2

$$R^2 = 1 - \frac{\sum_{j=1}^{n} \left(y_j - \hat{y}_j \right)^2}{\sum_{j=1}^{n} \left(y_j - \bar{y} \right)^2}$$

In the formula, y_j is the actual value of the jth sample point; $\bar{y}$ is the average of the actual values y of the sample points; $\hat{y}_j$ is the approximate value of the jth sample point; n is the number of sample points.

(2) Revised complex correlation coefficient R_{adj}^2

The sum of squares of regression bias is N_h:

$$N_h = \sum_{j=1}^{n} \left(\hat{y}_j - \bar{y} \right)^2$$

The corrected complex correlation coefficient R_{adj}^2 is:

$$R_{adj}^2 = \frac{\sum_{i=1}^{n} \left(y_j - \hat{y}_j \right)^2 / n - n_v - 1}{\sum_{j=1}^{n} \left(y_j - \bar{y} \right)^2 / n}$$

In the equation, n_v is the degree of freedom of the sum of squared regression deviations.
When conducting response surface methodology optimization, the closer the values of R^2 and R_{adj}^2 are to 1, the smaller the impact of errors and the better the fit of the response surface model.

2.3 Experimental Design

In the application of response surface methodology, the selection of random design variables is also an important part. At present, the commonly used selection methods for response surface optimization design include orthogonal design, uniform design, and center composite design.
Orthogonal design is an efficient design method for studying multiple factors and levels. It selects points with the characteristics of "uniform dispersion, neat comparability" from all experiments based on orthogonality for experimentation. Orthogonal design is an efficient and economical experimental design method that combines different factors and levels of experiments evenly and arranges them reasonably, which can greatly reduce the number of experiments. Generally, a suitable orthogonal experimental design scheme is searched on an orthogonal table based on the number of factors and levels. The orthogonal table can be denoted as $L_n(a^P)$, where L represents the orthogonal table, n represents the number of experiments, a represents the number of levels of factors, and P represents the maximum number of factors that can be arranged. Common orthogonal tables include $L_4(2^3)$、$L_8(2^7)$、$L_9(3^4)$、$L_{16}(2^{15})$、$L_{27}(3^{13})$、$L_{16}(4^5)$ etc.
Uniform design is a testing method that distributes uniformly throughout the entire experimental range. Uniform design only considers the sufficient "uniform dispersion" of sample points within the scope of the experiment, without considering "neatness and comparability", which can greatly reduce the number of experiments. However, the processing method for the results of uniform design experiments is regression analysis. Compared with orthogonal design, uniform design has better uniformity in experiments, making the experimental points more representative.
The central composite design combines the traditional interpolation node distribution method with orthogonal or uniform design, and is the most popular quadratic response test point design method. It is a factor design that expands the center point and a set of star shaped points, and has significant effects on estimating first-order and second-order polynomials. This article adopts the central composite design method to optimize the response surface methodology structure of the rotor shaft.

3 Example analysis

Import the three-dimensional models of the inner and outer rotor shafts into the finite element analysis software ANSYS Workbench, and establish finite element models of the inner and outer rotor shafts. The aerodynamic loads on the inner and outer rotor shafts caused by the rotor are shown in Table 1.

Table 1. Limiting load of inner and outer rotor shafts

Load item	Value
Lift of upper rotor	100000 N
Longitudinal force on the inner rotor shaft	+/-15000 N
Lateral force on the inner rotor shaft	-/+12000N
Rolling torque of inner rotor shaft	+/-102000 N•m
Pitch moment of inner rotor shaft	+/-107000N•m
Shaft torque of inner rotor shaft	-41000N•m
Lower rotor lift	100000 N
Longitudinal force on the outer rotor shaft	+/-15000N
Lateral force on the outer rotor shaft	+/-12000N
Rolling torque of outer rotor shaft	-/+102000N•m
Pitch moment of outer rotor shaft	+/-107000N•m
Shaft torque of outer rotor shaft	41000 N•m

Grid the inner rotor shaft, and the model after grid division is shown in Figure 1. Apply fixed constraints and limit loads on the inner rotor shaft to conduct static strength simulation analysis of the inner rotor shaft.

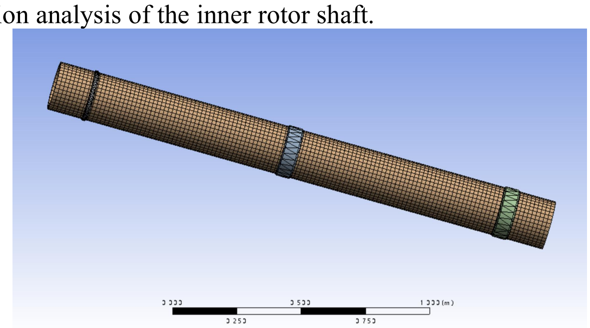


Figure 1. Grid division results of inner rotor shaft

The outer rotor shaft is meshed, and the meshed model is shown in Figure 2. Apply fixed constraints and limit loads on the outer rotor shaft to conduct static strength simulation analysis on the inner rotor shaft.

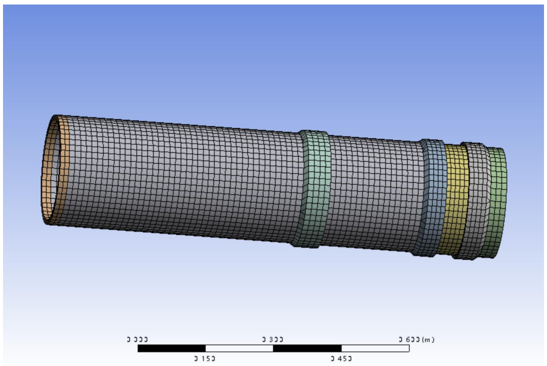


Figure 2. Grid division results of outer rotor shaft

The purpose of structural optimization is to minimize the weight of the structure and use the least amount of materials, while meeting the functional and mechanical performance requirements, in order to achieve the goal of the lightest weight of the structure. In the structure of the shaft, the main parameters that affect the weight of the shaft are the inner diameter and wall thickness of the shaft, so the inner diameter and wall thickness of the shaft are used as design variables. The range of values for the design variables of the inner and outer rotor shafts is shown in Tables 2 and 3. This article takes the maximum stress of the rotor shaft as the constraint condition and the weight of the rotor shaft as the objective function to conduct response surface optimization design of the rotor shaft structure.

Table 2. The range of values for the design variables of the inner rotor shaft

Optimization scope	Internal diameter d_1/mm	Wall thickness b_1/mm
Lower limit	160	9
Original value	168	11
Upper limit	175	15

Table 3. The range of values for the design variables of the outer rotor shaf**t**

Optimization scope	Internal diameter d_2/mm	Wall thickness b_2/mm
Lower limit	203.4	6.5
Original value	226	9.5
Upper limit	248.6	12.5

The experimental design method for optimizing the response surface of the rotor shaft adopts the center composite design method. The simulation results of the inner and outer rotor shafts are shown in Tables 4 and 5.

Table 4. Design Results of Inner Rotor Shaft Test

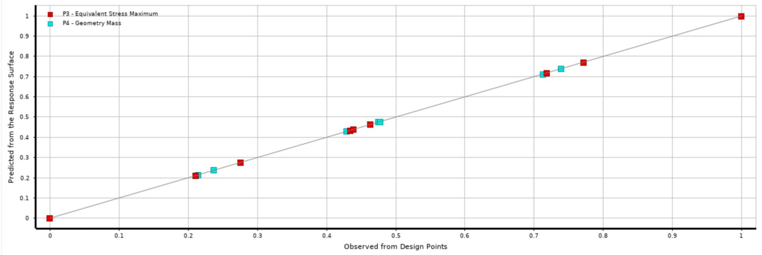
No	Internal diameter d_1/mm	Wall thicknessh_1/mm	Maximum stress/MPa	Weight of inner rotor shaft/kg
1	168	11	1167.7	100.21
2	151.2	11	1418.1	90.831
3	184.8	11	1050.3	109.59
4	168	9.9	1378.4	89.929
5	168	12.1	1002.1	110.62
6	151.2	9.9	1587.7	81.458
7	184.8	9.9	1189.6	98.399
8	151.2	12.1	1171.3	100.32
9	184.8	12.1	845.2	120.91

Table 5. Design results of outer rotor shaft test

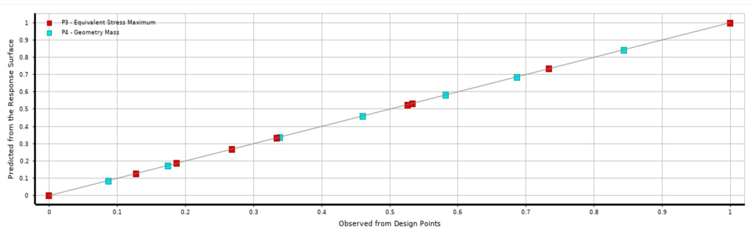
No	Internal diameter d_2/mm	Wall thicknessb_2/mm	Maximum stress/MPa	Weight of inner rotor shaft/kg
1	226	9.5	984.8	61.94
2	226	6.5	1356.4	43.87

3	226	12.5	792.4	80.45
4	203.4	9.5	1163.5	56.04
5	248.6	9.5	848.2	67.84
6	203.4	6.5	1603.9	39.65
7	203.4	12.5	923.4	72.88
8	248.6	6.5	1169.7	48.10
9	248.6	12.5	673.1	88.02

After completing 9 calculations, the obtained data was fitted with parameters to obtain a response surface model. In order to determine the accuracy of the fitted response surface model, it is necessary to compare the constraint variables and target variable values of the experimental design points with the values obtained based on the fitted response surface model. The relationship between the actual calculated values of the inner and outer rotor shafts and the predicted values obtained through the response surface model is shown in Figure 3. The stress values of the inner and outer rotor shafts are evenly distributed at both ends of the straight line y=x. Therefore, it can be seen that the response surface model obtained based on the response surface method can accurately represent the changes in stress and weight of the inner and outer rotor shafts. The fitted response surface model can be used to calculate the minimum weight point of the rotor shaft. The response surface models of the inner and outer rotor shafts are shown in Figures 4 and 5.

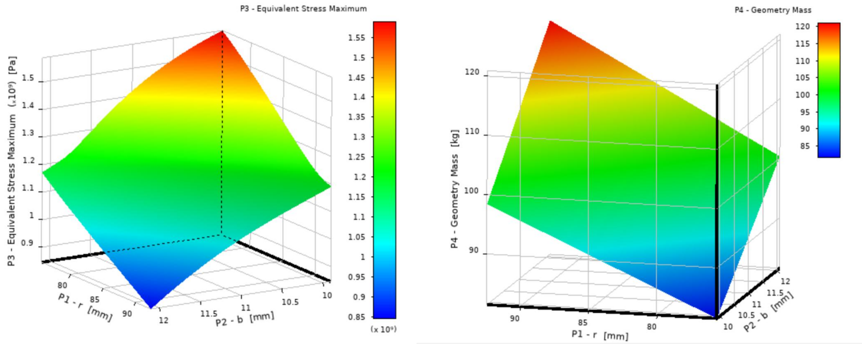


（a）Inner rotor shaft



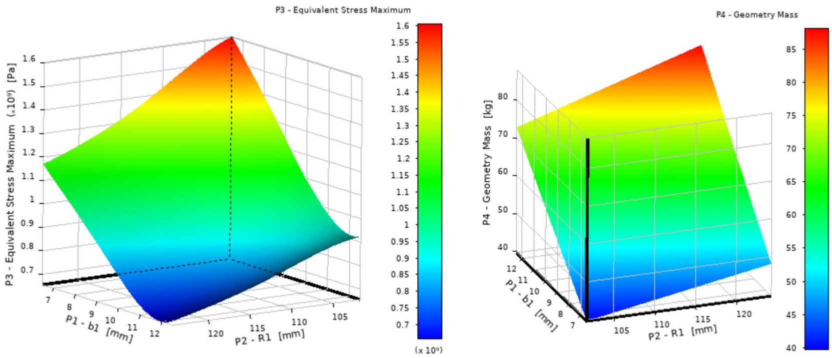
（b）Outer rotor shaft

Figure 3. Schematic diagram of optimized prediction of response surfaces for inner and outer rotor shafts - normalized comparison of observed values



(a)Stress response surface model (b)Weight response surface model

Figure 4. Response surface model of inner rotor shaft



(a)Stress response surface model (b)Weight response surface model

Figure 5. Response surface model of outer rotor shaft

Based on the fitted response surface model, the selection method is used to optimize the design of the inner and outer rotor shafts. The screening method is based on the response surface model to find the best design point among the given design points. The optimization accuracy is closely related to the number of selected design points. In this optimization, the number of design points is set to 3000. Under the premise that all optimization parameters meet the requirements, three sets of selectable optimization design points can be generated with the weight of the rotor shaft as the objective. The optimization design of the inner and outer rotor shafts of the transmission system resulted in the following optimization results.

(1)Optimization design results of inner rotor shaft

The three sets of candidate design points for the optimized inner rotor shaft are shown in Table 6. After comprehensive evaluation considering factors such as process and weight of the inner rotor shaft, the first set of candidate points is selected as the optimization result for the inner rotor shaft. Table 7 shows the stress and weight of the inner rotor shaft before and after optimization based on response surface methodology. According to Table 7, the stress on the inner rotor shaft increased by 1.65% before and after optimization, and the mass of the inner rotor shaft after optimization was 98.605kg, which decreased by 1.63% compared to the mass before optimization of 100.21kg.

Table 6. Optimization design results of inner rotor shaft

No	Internal diameter d_1/mm	Wall thickness b_1/mm	Maximum stress /MPa	Weight of inner rotor shaft/kg
1	180.384	10.147	1187	98.605
2	182.276	10.048	1187	98.612
3	179.598	10.187	1186.9	98.616

Table 7. Comparison of Optimization Results for Inner Rotor Shaft

No	Internal diameter d_1/mm	Wall thickness b_1/mm	Bending stress /MPa	Weight of inner rotor shaft/kg
Before optimization	168	11	1167.7	100.21
After optimization	180.384	10.147	1187	98.605
before and after comparison	—	—	1.65%	-1.63%

(2)Optimization design results of outer rotor shaft

The three sets of candidate design points for the optimized outer rotor shaft are shown in Table 8. After comprehensive evaluation considering factors such as process and weight of the outer rotor shaft, the first set of candidate points is selected as the optimization result for the outer rotor shaft. Table 8 shows the stress and weight of the outer rotor shaft before and after optimization based on response surface methodology. According to Table 9, the stress on the outer rotor shaft increased by 1.41% before and after optimization, and the mass of the outer rotor shaft after optimization was 59.14 kg, a decrease of 4.52% compared to the mass before optimization of 61.94 kg.

Table 8. Optimization design results of outer rotor shaft

No	Internal diameter d_2/mm	Wall thickness b_2/mm	Maximum stress /MPa	Weight of outer rotor shaft/kg
1	247.94	16.45	991.4	59.26
2	246.18	16.55	995.4	59.17
3	244.06	16.69	998.9	59.14

Table 9. Comparison of Optimization Results for Outer Rotor Shaft

No	Internal diameter d_2/mm	Wall thickness b_2/mm	Bending stress /MPa	Weight of outer rotor shaft/kg
Before optimization	226	9.5	984.8	61.94
After	244.06	16.69	998.9	59.14

optimization before and after comparison	—	—	1.41%	-4.52%

4 Conclusion

This article focuses on the rotor shaft of the coaxial reverse transmission system. Using response surface optimization method, a response surface optimization model based on ANSYS Workbench finite element simulation analysis software was established. Based on the fitted response surface model, the structural optimization design of the inner and outer rotor shafts was carried out. The comparison and analysis of the rotor shafts before and after optimization showed that the maximum stress of the rotor shaft before optimization was less than the allowable stress of the material, and the weight of the inner and outer rotor shafts decreased by 1.63% and 4.52% respectively, indicating the effectiveness of the structural optimization of the coaxial reverse transmission system rotor shaft based on response surface method.

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