Optimal iIMC-PD Double-Loop Control Strategy for Integrating Processes with Dead-Time

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Abstract. In this work, indirect internal model control – proportional derivative (iIMC-PD) double-loop controller is suggested to control the dynamics of integrating processes with dead-time. The inner loop consists of a PD controller which is responsible for the disturbance rejection. Its operational settings are determined using Routh-Hurwitz (RH) stability criteria. The outer loop comprises of the indirect IMC controller which is responsible for the set point tracking. The optimal value of both inner loop as well as outer loop tunables are obtained using a metaheuristic technique called the Equilibrium Optimizer (EO) with the objective of minimizing the Integrated Squared Error (ISE). Simulation results establish the satisfactory enhancement of the suggested control strategy over some of the recent works.

Keywords: Integrating process, dead-time, indirect IMC, equilibrium optimizer, integral squared error.

1 Introduction

Integrating processes are often encountered in chemical industries which include dynamics related to storage tanks, drying mechanism, distillation columns etc [1]. Due to their non self-regulation nature, the process output becomes unbounded at the advent of any disturbance or change at the plant input [2]. When these processes are modeled, they have at least one pole at the origin. Time delay is inherently present in these processes due to mechanical and thermodynamic constraints such as velocity lags, transportation loops, composition loops etc [3]. Due to these challenges, the integrating process requires dedicated control strategy as compared to the stable processes with dead time.

Proposals available at the literatures for controlling integrating process include Smith Predictor [2], IMC-PID controller [4] and IMC-PD controller [5]. IMC has fewer tunable parameters as opposed to some of the advanced control techniques and it provides good trade-off between performance and complexity of design [5]. The double-loop control strategy is increasingly getting popular. Several works in the literature are available where inner/outer-loop stabilization is achieved using PD/PI controllers, respectively. Li [6] designed a PI-PD control structure to stabilize a gasoline vapor pressure tower where the PI controller was designed by model predictive control method. Nema and Padhy [7] have optimized the PI and PD settings by Cuckoo Optimization algorithm. Onat [8] have adopted a graphical approach in which the PI-PD controller parameters were tuned by determining the centroid of the convex stabilizing area. Raja and Ali [9] used RH-stability criteria and moment matching techniques to design inner-loop PD and outer-loop PI controllers, respectively. Irshad and Ali [10] and Kaya [11] have obtained optimal PI-PD settings by using Particle Swarm Optimization (PSO).

The indirect-IMC (iIMC) design as reported by Verma and Padhy [12] is utilized to control the dynamics of delayed stable processes. However, its application for double-loop control on delayed integrating process remains to be seen. Equilibrium Optimizer (EO) has proved to be an effective algorithm [13] to obtain optimal controller settings for various applications including load frequency control [14] and unstable process control [15]. It has fared better than some of the contemporary algorithms on convergence and fitness measures. So, it is worth utilizing the algorithm for double-loop control of integrating process as well.

The present work aims to combine the advantages of iIMC approach and double-loop control scheme to control the integrating process with dead time. The inner-loop is stabilized with the proportional-derivative controller whose initial gains are chosen as per the Routh-Hurwitz (RH) stability criterion. The outer-loop is designed using the iIMC approach whose tuning parameters along with inner-loop derivative gain are obtained at the minimal Integral Squared Error (ISE) from the EO algorithm. The key contributions of this article can be summarized as follows:

- Design of novel iIMC-PD double-loop control scheme for integrating process with dead-time
- Inner-loop stabilization on RH stability criterion
- Application of EO to find optimal settings of tunable parameters

Rest of the sections are arranged in the following manner: Section 2 presents the theoretical background of iIMC approach. The suggested control scheme and EO are discussed in sections 3 and 4. Simulation results and related discussion is presented in section 5 followed by conclusion in section 6.

2 Theoretical background

Indirect Design Technique (IDT) as reported in [12] have given the following propositions:

Proposition A: Consider any stable process G(s) controlled by C(s) using an unity feedback structure shown in Figure 1(a). If the maximum sensitivity and phase margin of C(s)G(s) are M_1 & φ_1 and that of C(s)G(s-ψ) are M_2 & φ_2 respectively, then they satisfy the following conditions:

$$M_1 \leq M_2 \text{ and } \varphi_1 \geq \varphi_2 \ \forall\ \psi \geq 0$$
$$M_1 > M_2 \text{ and } \varphi_1 < \varphi_2 \ \forall\ \psi < 0$$

The shifting variable ψ behaves as the robustness parameter. ψ plays an important role in controller design which can be explained from the next proposition.

Proposition B: Let the controllers $C_1(s)$ & $C_2(s)$ been designed for controlling processes G(s-ψ) & G(s) respectively. The maximum sensitivities Ms_1 & Ms_2 corresponding to loop transfer functions $C_1(s)G(s)$ & $C_2(s)G(s)$ satisfy the following relationships:

$$Ms_1 \leq Ms_2 \ \forall\ \psi \geq 0$$
$$Ms_1 > Ms_2 \ \forall\ \psi < 0$$

This means that if the controller is designed for pole/zero shifted process in place of the original process, then the variation of ψ can help to achieve the desired level of robustness. The proof of the above proposition can be found in [12]. The following class of plant models are considered in this work:

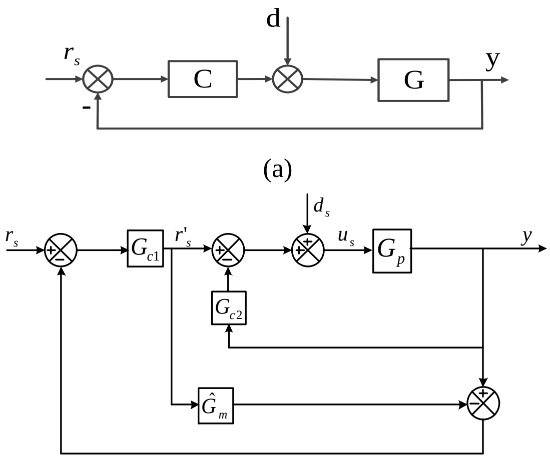
$$G_P(s) = \frac{K}{s} e^{-\theta s} \tag{1}$$

$$G_P(s) = \frac{K}{s(\tau s + 1)} e^{-\theta s} \tag{2}$$

Equation (1) represents integrating plus time delay (IPTD) while (2) represents second order integrating plus time delay (SOIPTD) process. K is the gain, τ is the time constant and Θ is the delay associated with the process.

3 Suggested control scheme

The suggested control scheme involves two controllers: G_{c1} in the outer-loop and G_{c2} in the inner-loop. G_{c2} is the PD controller responsible for the inner-loop stability and disturbance rejection. The outer-loop controller G_{c1} is the iIMC controller which is responsible for set-point following. In Figure 1(b), 'r_s' is the desired setpoint which the plant output 'y' must follow. Disturbance entering the process is denoted by 'd_s' whereas 'u_s' is the control effort which is given to the process 'G_p'. G_m and $\hat{G}_m$ are normal and frequency-shifted stabilized plant models which are discussed in the next subsections.



(b)

Fig. 1. (a) Unity feedback scheme (b) Suggested double-loop control scheme

3.1 Inner-loop control

G_{c2} is given by the following equation:

$$G_{c2}(s) = K_P(1 + \tau_D s) \tag{3}$$

From Figure 1(b), the inner closed-loop transfer function is given as

$$\frac{y}{r'_s}=\frac{G_p}{1+G_pG_{c2}}=G_m \tag{4}$$

where, G_m is the model of inner closed-loop transfer function to be used for designing G_{c1}. Using (3) and the IPTD plant model given by (1) into (4), we get

$$G_m(s)=\frac{Ke^{-\theta s}}{s+KK_P(1+\tau_D s)e^{-\theta s}} \tag{5}$$

Using the Pade's approximation $e^{-\theta s}\approx\frac{(1-0.5\theta s)}{(1+0.5\theta s)}$ in the denominator of (5), we get

$$G_m(s)=\frac{Ke^{-\theta s}}{s+KK_p(1+\tau_D s)\frac{(1-0.5\theta s)}{(1+0.5\theta s)}} \tag{6}$$

By selecting $\tau_D=0.5\theta$, one pole in the denominator gets cancelled. After rearranging the terms, we get

$$G_m(s)=\frac{Ke^{-\theta s}}{(1-0.5\theta KK_p)s+KK_p} \tag{7}$$

Proceeding on similar lines by substituting (3) and (2) into (4), the inner-loop transfer function for SOIPDT process can be obtained as follows:

$$G_m(s)=\frac{Ke^{-\theta s}}{\tau s^2+(1-0.5\theta KK_p)s+KK_p} \tag{8}$$

Applying RH criteria on (7) and (8), we get $(1-0.5\theta KK_p)>0$ and $KK_p>0$. Accordingly, the stable range of K_P is obtained as $0<K_p<\frac{1}{0.5\theta K}$ for IPTD and SOIPTD processes with $\tau_D=0.5\theta$.

3.2 Outer-loop control

$G_m(s)$ for IPTD process is given by (7). It can be re-written as

$$G_m(s)=\frac{Ke^{-\theta s}}{a_1 s+a_0} \tag{9}$$

where, $a_1=(1-0.5\theta KK_p)$ and $a_0=KK_P$. Applying the iIMC approach, $G_m(s-\psi)$ can be given as

$$\hat{G}_m(s)=G_m(s-\psi)=\frac{\hat{K}e^{-\theta s}}{\hat{a}_1 s+\hat{a}_0} \tag{10}$$

Similarly, for SOIPTD $G_m(s)$ and $G_m(s-\psi)$ can be given as

$$G_m(s)=\frac{Ke^{-\theta s}}{a_2 s^2+a_1 s+a_0} \tag{11}$$

$$\hat{G}_m(s)=G_m(s-\psi)=\frac{\hat{K}e^{-\theta s}}{\hat{a}_2 s^2+\hat{a}_1 s+\hat{a}_0} \tag{12}$$

where, $\hat{K}=Ke^{\psi\theta}$, $\hat{a}_2=a_2$, $\hat{a}_1=\left(a_1-2\psi a_2\right)$ and $\hat{a}_0=\left(a_2\psi^2-a_1\psi+a_0\right)$. $\hat{G}_m(s)$ can be decomposed as:

$$\hat{G}_m(s)=\hat{G}_m^-(s)\times\hat{G}_m^+(s) \tag{13}$$

where $\hat{G}_m^-(s)$ is the invertible component while $\hat{G}_m^+(s)$ is the non-invertible component of $\hat{G}_m(s)$. Now, the iIMC controller G_{c1} is designed as

$$G_{c1}(s)=\left[\hat{G}_m^-(s)\right]^{-1}G_f(s) \tag{14}$$

where $G_f(s)$ is a low pass filter that has a tuning parameter λ. The final expression of G_{c1} is given as

$$G_{c1}(s)=\left(\frac{\hat{a}_2s^2+\hat{a}_1s+\hat{a}_0}{\hat{K}}\right)\times\frac{1}{(\lambda s+1)^n} \tag{15}$$

n is the order of $G_f(s)$ which depends on that of $\hat{G}_m(s)$.

4 Equilibrium optimizer

EO is a recent physics-inspired metaheuristic algorithm mimicking the dynamic movements of volume mass models towards equilibrium states [13]. The model particles act as search agents with their concentration indicating positions. The particles keep on updating their concentrations till they reach equilibrium state or optimal solution. Steps involved in EO are:

Step-1: Initialization search agents

The initial concentration of particle is constituted with uniform randomness in search space given by:

$$C_k^{in}=C_{\mathrm{m}}+rand_k(C_{\mathrm{m}}-C_{\mathrm{n}}) \quad k=1,2,3.......n \tag{16}$$

where C_k^{in} is the starting concentration vector of kth search agent, C_m and C_n denote maximum and minimum value of vector, $rand_k$ adds randomness in the range (0,1).

Step-2: Pool particles

Search agents behave as pool particles which are used to construct equilibrium pool vector given by:

$$\overline{C}_{p,eqb=}\{\overline{C}_{p(1)},\overline{C}_{p(2)},\overline{C}_{p(3)}........,\overline{C}_{p(ave)}\} \tag{17}$$

Step-3: Propagation rate.

Propagation rate provides improved solution by enhancing the exploitation phase. It is expressed by the following first-order exponential equation:

$$\overline{P}=\overline{P}_0\ e^{-\vec{k}(t-t_0)} \tag{18}$$

where $\overline{P}_0$ is the initial propagation rate and $\vec{k}$ is the depreciation constant.

Step-4: Exploration and exploitation

Position of particle is updated in exploration phase which is given by:

$$\vec{C} = \vec{C}_p + (\vec{C} - \vec{C}_p).\ \overrightarrow{F} + \frac{\vec{P}}{\sigma\vec{V}}(1 - \vec{G}) \tag{19}$$

where $\vec{C}$ is the pool concentration volume (V) and σ is the crossover rate. Finally, the exploitation function G is given as:

$$\vec{G} = \eta_1 sign(\vec{r} - 0.5)[e^{-\vec{\sigma}\rho_{iter}} - 1] \tag{20}$$

where η_1 is a constant. ρ_{iter} is iteration function given by

$$\rho_{iter} = \left(1 - \frac{\rho}{\rho_{\mathrm{m}}}\right)^{\left(\eta_2 \frac{\rho}{\rho_{\mathrm{m}}}\right)} \tag{21}$$

ρ and ρ_m represent current and maximum iterations while η_2 is constant. EO flowchart is presented in Figure 2.

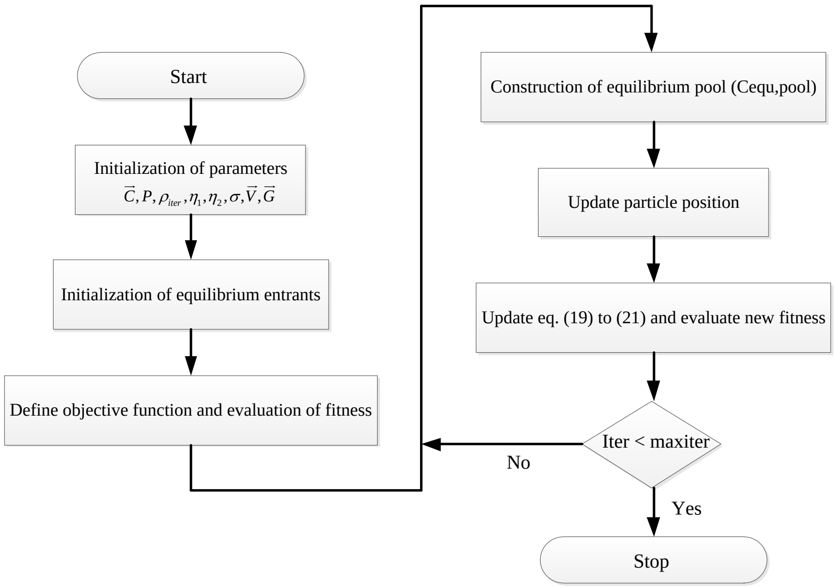


Fig. 2. Flowchart of EO

5 Results and discussion

The proposed control strategy is studied for IPTD and SOIPTD plant models. The controller gains with range constraints were utilized by EO. The performance measures considered for comparison are ISE and IAE whose expressions are given as:

$$ISE = \int_0^t e^2(t)dt = \int_0^t [r_s(t) - y(t)]^2 dt \tag{22}$$

$$IAE = \int_0^t |e(t)|\,dt = \int_0^t |r_s(t) - y(t)|\,dt \tag{23}$$

The initial population or search agents considered is 10 while keeping the maximum number of iteration as 20. The simulation is run with a step disturbance of magnitude '-1' given to the process at t=15 sec and t=60 sec for examples 1 & 2 respectively to analyse the regulatory behaviour of the system. The values of optimized controller gains are given in Table-1. To analyse robust operation, the system parameters (K, τ, Θ) were perturbed by 20% for same controller settings. The comparison of error functions for nominal and perturbed cases are presented in Tables 2 & 3.

Table 1. EO optimized controller settings

Example	**Parameters**		
	$\mathbf{K_p}$	$\boldsymbol{\lambda}$	$\boldsymbol{\psi}$
Example-1	1.5423	1.2210	0.0106
Example-2	3.2147	0.4801	0.0120

5.1 Example-1

Consider an IPTD process given by

$$G_P(s) = \frac{e^{-0.5s}}{s} \tag{24}$$

Ajmeri and Ali [16] have designed a parallel control structure for an IPTD process $G_p(s) = e^{-0.5s}/s$ with the following controller settings: K_{c1} = 1.4966, T_{d1} = 0.2378, λ=0.3280, τ=0.6025, K_{c2} = 1.6605, T_{i2} = 2.0575 and T_{d2} = 0.1770. To analyze the regulatory response, d=-1 is introduced at t=15 sec. Achieved closed-loop outputs with control signals are presented in Figure 3. The proposed controller has three tunable parameters (λ, K_P & ψ). Now, using the relations established in sections 3.1, we get τ_d=0.25 and range of K_p as (0,4) for stable operation. The range of λ and ψ is chosen (1,2) and (0.01,0.1) based on sensitivity considerations [15]. The design method of Ajmeri and Ali [16] yields considerably large overshoot during the servo action. Though the settling time of the suggested scheme is slightly higher, it yields a much lower peak value than that of [16] for a regulatory response. The control efforts required during set-point change are also higher for [16] which is not desirable. From Figure 3, it is obvious that the suggested control strategy delivers smoother control action, better setpoint following and disturbance elimination even in perturbed cases. The suggested scheme also shows improved IAE and ISE performance measures for servo and regulatory responses as given in Table 2.

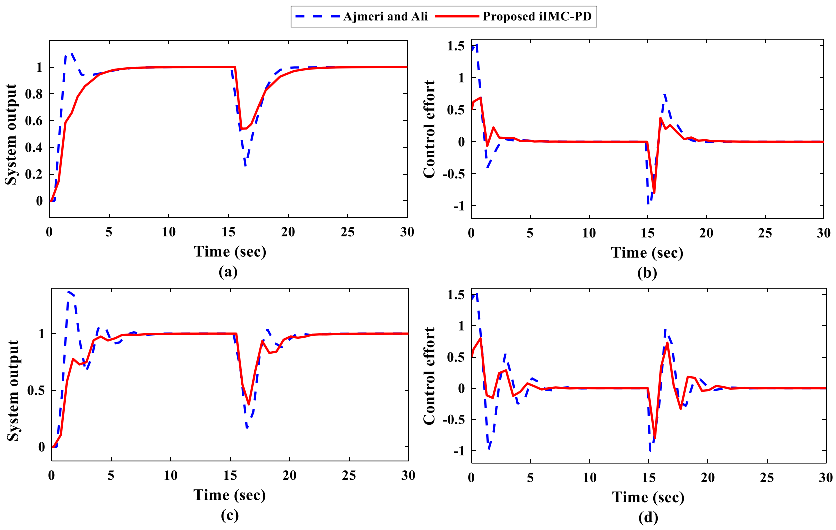


Fig. 3. ((a) Output responses in the nominal case, (b) Output responses in the perturbed case, (c) Control efforts in the nominal case, (d) Control efforts in the perturbed case)) for example-1

Table 2. Performance measure comparison for example-1

Method	**Nominal**		**Perturbed**	
	ISE	**IAE**	**ISE**	**IAE**
Ajmeri and Ali	1.519	2.910	1.685	2.929
Proposed	1.460	2.875	1.525	2.874

5.2 Example-2

Irshad and Ali [10] have studied the following ISOPTD process model:

$$G_P(s) = \frac{0.2e^{-s}}{s(4s+1)} \tag{25}$$

The controller settings of [10] are: K_p = 2.455, T_d = 3.508, K_c =1.491 and T_i =3.568. To analyze the regulatory response, d=-1 is introduced at t=50 sec. Closed-loop outputs with control signals are presented in Figures 4(a) and 4(b) for the unperturbed model. Perturbed system responses and control signals for $\pm 20\%$ perturbation (+20 % in K and Θ; –20 % in τ) are presented in Figures 4(c) and 4(d), respectively. The proposed controller's tunable parameters (λ, K_c & ψ) operating ranges are once again obtained using the relations established in section 3.1. We get τ_d=0.5 and range of K_p as (1,10) for stable operation. The range of λ and ψ is chosen as (0,1) and (0.01,0.1) respectively. Though the servo response of [10] is comparable with the proposed method (refer to Figure 4(a)), it shows a larger undershoot during regulatory action. In practical scenarios, regulatory action is more vital than servo action which makes the proposed method more preferable [9]. Since perfect plant models are not possible in practice, the controller design needs to be robust to perturbations in plant parameters. The control efforts of [10] show sudden spikes as seen in Figure 4(d) which is not desirable in practical case.

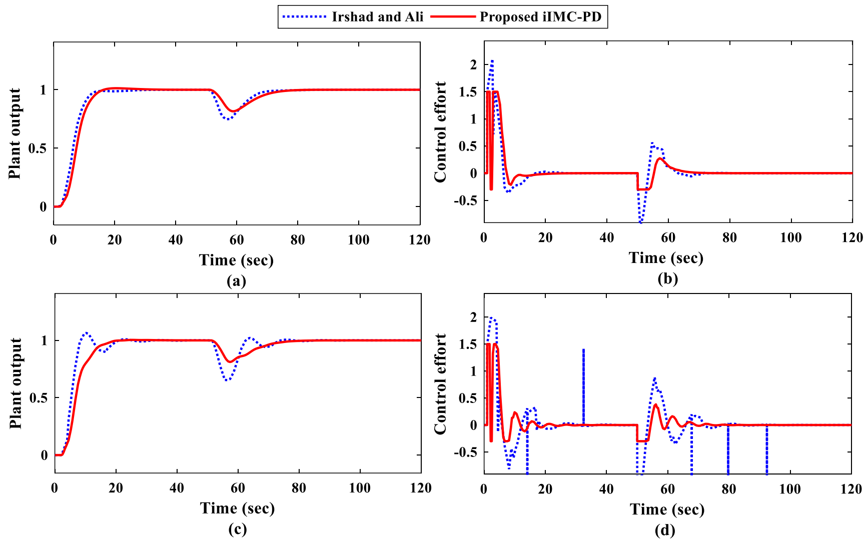


Fig. 4. ((a) Output responses in the nominal case, (b) Output responses in the perturbed case, (c) Control efforts in the nominal case, (d) Control efforts in the perturbed case)) for example-2

Table 3. Performance measure comparison for example-2

Method	Nominal		Perturbed	
	ISE	IAE	ISE	IAE
Irshad and Ali	5.671	9.268	5.240	8.713
Proposed	5.440	8.982	5.088	8.790

6 Conclusions

In this work, an optimal indirect IMC-PD double-loop control strategy is proposed for integrating processes with dead time. The inner-loop is controlled for the disturbance rejection by the proportional-derivative controller complying the Routh-Hurwitz RH stability criterion. The outer-loop servo controller is designed using the indirect-IMC approach. The optimal settings of tunable parameters of both the control-loops are obtained with the objective of minimizing integral squared error from the EO algorithm. The suggested strategy yields enhanced servo and regulatory response as compared to the some of the prevalent double-loop strategies. Future research can be directed towards optimizing indirect IMC tuning rules for the double-loop control strategy with guaranteed robustness.

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