EasyChair Preprint

№ 4294

Analysis of Problem Solving Difficulties at Limits of Sequences

Isnani, S.B. Waluya, Rochmad, Dwiyanto and T.S.N. Asih

September 29, 2020

Analysis of Problem Solving Difficulties at Limits of Sequences

Isnani[1], S B Waluya[2], Rochmad[2], Dwiyanto[2], and T S N Asih[2]

[1]Departement Mathematics Education Universitas Pancasakti Tegal Indonesia
[2]Departement Mathematics Education Universitas Negeri Semarang Indonesia

Email: isnani@upstegal.ac.id and isnani.ups@gmail.com

Abstract. Students in studying Real Analysis have difficulty. Sequences and Sequences are difficult items in Real Analysis. Likewise, one of the sub materials in the sequence and series, namely the sequence limit. Difficulties in learning the concept of limits include difficulties in understanding the definition and visualization of the limit concept. The obstacle to thinking of these students is shown from the difficulties of the students above. This research is a descriptive qualitative research with research subjects, namely students who take the Real Analysis course in the 2019/2020 academic year in the Mathematics Education Study Program of Pancasakti University, Tegal. Then interviewed were six students who were two high category students, two medium category students, and two low category students. When solving the line limit problem, students experience difficulty thinking due to the lack of initial analysis and the lack of student analogy skills. When the procedure is checked again, students have difficulty thinking too, this is due to the lack of mathematical connection skills of students. To overcome these student difficulties, the lecturer must provide Scaffolding.

1. Introduction

The Real Analysis course is one of the courses in accordance with the objectives of mathematics in Higher Education. The Real Analysis course is a compulsory subject in the Mathematics Education Study Program. Real analysis course is a very important subject. This course is axiomatic deductive. Most mathematics education students consider the Real Analysis course a difficult subject,[1]. Because the material in these courses is abstract, so students experience difficulties. The Real Analysis course is the first course that students learn to practice reasoning and prove mathematics formally. The difficulties encountered by students are lack of mastery of proof techniques, lack of understanding of premises, inability to understand the use of definitions, theorems and existing problems, lack of mastery of writing sentences in language and mathematical notation, and lack of understanding of material requirements, [3].

One of the materials that are considered difficult in the Real Analysis course is the material of Sequences and Series, [2]. In the material of sequences and series, the point is to prove a convergent sequence and a convergent series. However, in proving the convergent sequence the theorem used varies greatly, its use is in accordance with the pattern or characteristics of the sequence. This condition makes students confused about the exact theory that will be used in proving the next theorem and the problem. One of the sub-materials of Sequences and Series, namely Limit of Sequences. [4] Limit of Sequences is a sub material that has a high difficulty level. Because there are many definitions and theorems in the sequence limits. This definition and theorem is widely used in its application. Considering that students consider the Barisan limit to be a difficult sub-material, the lecturer provides a solution, namely by providing Scaffolding.

Scaffolding is an interaction process by providing assistance or guidance to students by lecturers or friends to understand knowledge or skills that cannot be achieved without assistance. Scaffolding is one of the strategies that teachers can use in teaching and developing students' abilities [5]. In learning mathematics Scaffolding is an aid to solving problems, as well as helping to build concrete mathematical concepts and increase students' self-confidence. Scaffolding can be given by a discussion process between students and lecturers, as well as between students and students. Saffolding used in this study refers to research [6], namely level 1 (*environmental provisions*), level 2 (*explaining, reviewing, and restructuring*), and level 3 (*developing conceptual thinking*).

The purpose of this study is to describe the analysis of the difficulties of students of Mathematics Education at Pancasakti University of Tegal in the sub-material of the line limit, to describe the provision of appropriate scaffoding to overcome student difficulties in solving problems in the limit line material.

2. Method

2.1 Types and Research Approaches

The approach used in this study is a qualitative approach, according to [7] a qualitative approach is an approach that builds knowledge. S The research subjects were 61 students of the Pancasakti Tegal University Mathematics Education 2019/2020 who took Real analysis courses. Research data obtained from the results of observations, tests and interviews. The preliminary test was carried out at the beginning of the study to obtain a description of the student's initial abilities, so that an overview of the student's difficulties could be obtained. Then scaffolding is prepared to obtain the next information, maybe there is different information. So that the research does not expand and is still within the boundaries of the area, a scaffolding sheet is made, before scaffolding is done. Based on the results of the initial ability test and lecturer observations while teaching the Real Analysis subject in the 2019/2020 Academic Year, the research subjects were taken. Then the six students were determined, consisting of two students with high initial ability, two students with medium initial ability, and two students with low initial ability. Then the six students were determined, consisting of two students with high initial ability, two students with medium initial ability, and two students with low initial ability. If after scaffolding the student is able to correct the answer, then the analysis will be carried out. However, if the result has not been improved or cannot improve the Scaffolding, then Scaffolding will be given.. If after scaffolding the student is able to correct the answer, then the analysis will be carried out. However, if the result has not been improved or cannot improve the Scaffolding, then Scaffolding will be given.

2.1 Research Instruments

The main instrument in qualitative research is the researcher. Planners, implementers, data collectors, analysts, data interpreters, and also as reporters of research results are carried out by researchers. Test questions are a supporting instrument used by researchers. Test questions are given to students to get initial ability data and describe the student's difficulties in solving the limit line problem before obtaining scaffoding from the researcher.

Scaffolding sheets are another instrument used by researchers. The distribution of scaffolding sheets aims to analyze the difficulty of solving problems at the limit of the line as experienced by students, thus helping and directing students to correct the errors they have experienced. The scaffolding guide sheet contains questions, directions, and statements by researchers to students. The causes of these errors will be investigated by asking the questions in question. It is hoped that the errors experienced by students can be overcome by giving Scaffolding. Some of the scaffolding carried out can be seen in Table 1.

Table 1 Scaffolding Activities

Scaffolding Activities	Activities performed
Environmental	Make assignment sheets, prepare another picture when students

Provisions	do not understand the problem verbally
Explaining	Students must explain the definition of sequence real numbers according to what is already known, then students must read the problem repeatedly on the assignment sheet that has been given, then direct and explain to students in order to understand the problem
Reviewing	Students must reflect on the answers has been found so that students can find out their mistakes then students must correct them
Restructuring	Provide directions to students in the form of questions so that students can find existing problems, then students can answer them again with a better design
Developing Conceptual Thinking	Asking students to solve problems in other alternative ways, giving directions in the form of questions that can make students find other concepts related to existing problems.

3. Results and Discussion

Based on the results of the test questions and interviews, it was found that students had difficulty solving problems in the Limit Barisan sub-material. Student difficulties in solving problems occur when understanding the concept of the problem, preliminary analysis, carrying out evidence or solving, and examining the proving process and the results of the completion. The difficulties experienced by students include difficulty thinking due to the lack of students' initial analytical knowledge, difficulties due to the lack of students' logical abilities, and difficulties due to the lack of students' mathematical connection skills.

Difficulty thinking due to the lack of knowledge of the student's initial analyst occurs at the stage of understanding the concept of the problem, planning completion, and carrying out proof / completion. Students' thinking difficulties and types of initial analysis knowledge can be seen in Table 2.

Table 2 Types of students' initial abilities and thinking difficulties

Initial ability type	Student Thinking Difficulties	The type of scaffolding done
Bijective Function	Students forget the injective and surjective concepts	Reminiscent of the concept of functions, injective and surjective functions
Infinite function limit	Students have difficulty solving the limit of the infinite function on the division	Reminiscent of the concept of infinite limits when studying calculus
Real number sequence definition	Students still use arithmetic sequences and geometric sequences	Recalls the definition of real number sequences and provides some examples
Definition of sequence limits	Students forget the definition of sequence limits and cannot explain	Recall the definition of sequence limits with visualization and provide

		examples of problems
Convergent sequence definition	Students forget the definition of a convergent sequence and cannot visualize it	Recall the definition of convergent sequence with visualization and provide some sample problems
The Pinch Theorem	Students forget to solve the problem with the pinch theorem	It reminds you of the wedge theorem and gives some example problems
Row ratio	Students forget the concept of sequence ratio	Recalls the concept of sequence ratios and provides some sample problems

The difficulty in thinking of students due to the lack of logical ability of students occurs at the planning stage of completion, proving / completing, and re-examining the process of proof and solution. Students' thinking difficulties and the types of logic abilities can be seen in Table 3.

Table 3 Types of students' mathematical logic abilities and thinking difficulties

Logic type	Description of students' thinking difficulties	The type of scaffolding done
Bijective Function	Students have difficulty in surjective, think that the domain code is inexhaustible	Reminding again about the function, if it is surjective then the domain code runs out
Convergent and Divergent Sequences	Students forget the definition of Convergent and Divergent sequences	Reminiscent of the definition of convergent and divergent sequences with visualization
Determination (K)	Students cannot find the K value as well as in changing the form of inequality is wrong	Remind students and students to correct it according to the definition in determining (K) and students to solve inequality problems in fractions
The Pinch Theorem	Cannot prove the use of the fix theorem	Reminds back to using the fixed theorem and gives some examples of problems and their solutions
Row Ratio	Students forget to determine the comparison of the ranks	Recalling the Sequence ratio theorem, giving examples of problems and solutions

Then the difficulty of thinking due to the lack of mathematical connection skills of students occurs at the stage of completing and re-checking procedures and solutions. The difficulty of students' mathematical connection ability in succession can be seen in Table 4

Table 4 Types of students' mathematical connection ability and thinking difficulties

Mathematical Connection Types	Description of Student's difficulty in thinking	The type of scaffolding done
Bijective Function	Student incorrectly linked the domain and result area	Remind students about functions then about inverse functions
Convergent and Divergent Sequences	Students cannot make a pattern of a sequence and their conclusions are wrong	Remind members of the line one by one to form a line pattern
Limit of Sequences	Students are wrong in determining the concept of sequence limits	Reminiscent of the definition of sequence limits in real analysis, as well as with the initial analysis to determine the value of (K)
Proof of Limit	Students cannot use the use of limit definitions in solving problems	Reminded the definition of limit and asked students to improve it
Row ratio	Students cannot determine the rank comparison	Reminiscent of the sequence ratio theory and students to correct the answer

Students have difficulty thinking because there are seven initial abilities in table 2. Students' initial ability in the proofing process is very necessary. [8] Initial analysts are needed in the proving process because they can develop students' abilities in the proof process so that students can understand and validate arguments from mathematical statements. [9] Mathematical concepts are necessary for preliminary analytical knowledge because they can give meaning to the concept definition and related formal solutions. Procedural analysis needs to be done in the initial analysis because with procedural travel students can understand the technique of proof [10]. Initial analysis is very helpful in the evidentiary process, including by building graphic arguments then building symbol-verbal evidence based on these graphical arguments. [11].

It is difficult for students to think in solving the problem of line limits, because of their lack of mathematical logic skills. These difficulties in Table 3 are found to be five. In the process of proving the need for logical ability which has the consequence that the perfecting of the idea of equality will be obtained, [12]. Mathematical logic skills attempt to place all mathematics on the basis of set theory and classical logic [13]

Then it is difficult to think about solving the sequence limit problem because of the lack of mathematical connection skills. [14] Mathematical connection is the ability of students to use related ideas in mathematics, connections with other disciplines and apply mathematical ideas in the context of everyday life. They need mathematical connection skills in learning related mathematical concepts, [15]. Mathematical connection is the most important part that needs to be done in every level of education. Mathematical connection means the capacity above the information provided, with a critical attitude to evaluate something and having metacognitive awareness and problem solving skills, [16].

In real analysis it is necessary to make connections, as the ability to relate the proving task with definitions, theorems, multiple representations, and examples from the current where a student is, and the possible previous experiences of previous material. Within this category, it is necessary to

consider making connections: between definitions / theorems, between representations, and between examples. Each of the subcategories is described below. [17] Between Definitions / Theorems To improve the ability to make connections, students must use the definitions / theorems previously discussed in possible terms, from other materials.

Scaffolding is expected to play an important role in developing students' thinking process skills during learning, [18]. Scaffolding is very helpful for lecturers in mapping the conceptual development of students' mathematical thinking so that lecturers are better able to make informed instructional decisions, [19]. With scaffolding, lecturers can identify the relevance of student reasoning for both mathematical purposes and students' understanding of mathematical ideas related to these goals, [20]. scaffolding in mathematics learning is an aid to solving problems, as well as helping to build concrete mathematical concepts and increase students' self-confidence, [21].

4.Conclusions

Based on the results of the study, it was found that students 'difficulties in solving the limit lines were due to the lack of students' initial abilities, logical abilities, and mathematical connection skills. Lack of initial skills because students cannot understand the problem, and do not find ideas in solving problems. Lack of mathematical connection skills in Karen students, they do not understand the problem in solving / proofing, and do not check answers again and do not understand the evidentiary procedures used. Lack of mathematical logic skills due to lack of understanding of the problem, lack of completion plan at the time of completion.

To overcome the above difficulties, one of the steps taken is by giving Scaffolding. The given scaffolding starts from the level of environmental provisions or prepares another picture when students do not understand the initial problems. Explaining or providing explanations. Reviewing or reflecting on answers and improving work results. Restructuring is a question or direction to find answers and answer back with a better design. Meanwhile, the developing conceptual thinking stage or looking for other alternatives to solve problems and provide direction in order to find other concepts related to the problem.

References

[1] Isnani, Waluya S B, Rochmad, Wardono 2020a Analysis of mathematical creativity in mathematics learning is open ended *Journal of Physics Conference Series* **1511** 012102

[2] Isnani I, Waluya S B, Rochmad R, Sukestiyarno S, Suyitno A and Aminah N 2020b How is Reasoning Ability in Learning Real Analysis? *ICASSETH 2019* 253-256 Atlantis Press.

[3] Moore R C, 1994 Making Transition to Formal Proof," Journal of Educational Studies in Mathematics **27** 249-266

[4] Isnani I, Utami W B, P Susongko, Lestiani HT 2019 Estimation of college students' ability on Real Analysis course using Rasch model. *Research and Evaluation Education*, ***5*** 2, 1-7.

[5] BIKMAZ FH, ÇELEB Ö., Aslıhan, ATA, Eren, Ö. ZER, SOYAK, Ö, REÇBER H 2010 Scaffolding strategies applied by student teachers to teach mathematics *The International Journal of Research in Teacher Education*, **1** 3, 25-36.

[6] Anghileri J 2006 Scaffolding practices that enhance mathematics learning. Journal of *Mathematics Teacher Education*, ***9*** 1, 33–52.

[7] Cresweell J W 2014 *Research Design Qualitative Quantitative and Mixed Meethod Approach*.

[8] David E J and Zazkis D 2020. Characterizing introduction to proof courses: a survey of US R1 and R2 course syllabi. *International Journal of Mathematical Education in Science and Technology*, ***51*** 3 388-404.

[9] Weber K 2004 Traditional instruction in advanced mathematics courses: A case study of one professor's lectures and proofs in an introductory real analysis course. *The Journal of Mathematical Behavior*, ***23*** 2 , 115-133.

[10] Weber, K. (2005). A procedural route toward understanding aspects of proof: Case studies from real analysis. *Canadian Journal of Math, Science & Technology Education*, ***5*** 4, 469-483.

[11] Zazkis D Weber K Mejía-Ramos J P 2016 Bridging the gap between graphical arguments and verbal-symbolic proofs in a real analysis context. *Educational Studies in Mathematics*, ***93*** 2, 155-173.

[12] Morales J A C, Zilber B 2018 Logical perfection in mathematics and beyond. *arXiv preprint arxiv:1803.04909*.

[13] Butler R W 2009 Formalization of the integral calculus in the PVS theorem prover. *Journal of Formalized Reasoning*, ***2*** 1 1-26.

[14] Asfaroh, H., & Ekawati, R. (2019). Students Mathematical Connection in Problem Posing. *MATHEdunesa*, *8*(2).

[15] Amin M S B, Kartono K and Dewi N R 2020 Connected Mathematics Ability Seen from Student Cognitive Style on STAD–Peer Tutoring Learning Model. *Unnes Journal of Mathematics Education Research*, ***9*** 1, 93-99.

[16] Rohendi D , Dulpaja J 2013 Connected Mathematics Project (CMP) model based on presentation media to the mathematical connection ability of junior high school student. *Journal of education and practice*, *4*(4).

[17] Savic M, Karakok G, Tang G, El Turkey H and Naccarato E 2017 Formative assessment of creativity in undergraduate mathematics: Using a creativity-in-progress rubric (CPR) on proving. In *Creativity and giftedness* 23-46 Springer Cham.

[18] Angeli C, & Valanides 2020 Developing young children's computational thinking with educational robotics: An interaction effect between gender and scaffolding strategy. *Computers in Human Behavior*, *105*, 105954.

[19] Bywater J P , Chiu J L , Hong J , & Sankaranarayanan V 2019 The Teacher Responding Tool: Scaffolding the teacher practice of responding to student ideas in mathematics classrooms. *Computers & Education*, ***139***, 16-30.

[20] Andrews-Larson C, Mc Crackin S, & Kasper V 2019 The next time around: scaffolding and shifts in argumentation in initial and subsequent implementations of inquiry-oriented instructional materials. *The Journal of Mathematical Behavior*, ***56***, 100719.

[21] Wahyudi W, SB,Waluya, Rochmad, & Suyitno H 2018 Assimilation and Accommodation Processes in Improving Mathematical Creative Thinking with Scaffolding According to Learning Style. In *Journal of Physics: Conference Series* (Vol. 1097).