Fuzzy Logic and Fuzzy Reasoning with Two Fuzzy Membership Functions Based on Belief and Disbelief

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Abstract

Zadeh[29] defined fuzzy Sets for Uncertain Information with single Fuzzy membership function A = $\mu_A(x)$, where A is Fuzzy Set and x Є X. In this paper, the Fuzzy set is defined by A= { $\mu_A^{Belief}(x)$, $\mu_A^{Disbelief}(x)$} with the two Fuzzy membership functions $\mu_A^{Belief}(x)$, $\mu_A^{Disbelief}(x)$ based on Belief and Disbelief. The Fuzzy Set with two Fuzzy membership functions will give more evidence. Fuzzy Logic and Fuzzy reasoning are studied based on the two Fuzzy membership functions using the Fuzzy Modulations. Fuzzy Certainty Factor is defined with the difference of Belief Fuzzy Membership Function and Disbelief Fuzzy Membership Function to compute the conflict of evidence in Uncertain Information.

Key words: Fuzzy Membership Functions, Fuzzy Logic, Fuzzy Reasoning, Fuzzy Modulations

1 Introduction

Different methods are proposed to deal with incomplete, inconsistent, inexact and uncertain information. Fuzzy logic[33] Bayesian Statistics[4] , Dempster-Shafer Theory [5,19] , Certainty Factor [3] and Fuzzy Statistics [30] are proposed to deal with Uncertain Information's. Zadeh[29] defined Fuzzy set single membership function and formulated Fuzzy Logic to deal with uncertain information. The Fuzzy Set with two membership functions will give more evidence to deal with the uncertain information. The Many-Valued Logic[] is considered to discuss the Fuzzy Logic with two membership functions.

Fuzzy Set is defined by two Fuzzy membership functions based on "Belief and Disbelief'. The Fuzzy Logic and Fuzzy Reasoning are studied based on the two membership functions. The Fuzzy Certainty Factor is defined by the difference between "Belief" and "Disbelief " membership functions to make as single membership function.

. Fuzzy Modulations is a type of Knowledge representation for Fuzzy propositions [24]. These Fuzzy modulations are used to study Fuzzy Logic and Reasoning for two membership functions for understanding.

The brief introduction of Fuzzy Logic and Fuzzy Reasoning is given as follows to understand the above concepts.

2 Fuzzy Logic and Fuzzy Reasoning

Zadeh[29] has introduced Fuzzy set as a model to deal with imprecise, inconsistent and inexact information. Fuzzy set is a class of objects with a continuum of grades of membership.

The Fuzzy set A of X is characterized by its membership function $A = \mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x): X \rightarrow [0, 1], x \in X$, where X is Universe of discourse.
or
$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$, "+" is union

For example, Consider the Fuzzy proposition "x has Cold" .

The Fuzzy set 'Cold" is defined as

$\mu_{Cold}(x) \rightarrow [0, 1], x \in X$
where Cold = { $0.8/x_1 + 0.6/x_2 + 0.4/x_3 + 0.6/x_4 + 0.75/x_5$}

For instance "Rama has Cold" with Fuzziness 0.8

Fuzzy logic is defined as combination of Fuzzy sets using logical operators. Some of the logical operations are given below

Suppose A, B and C are Fuzzy sets, and the operations on Fuzzy sets are

$A \vee B = \max(\mu_A(x), \mu_B(x)\}$	Disjunction
$A \wedge B = \min(\mu_A(x), \mu_B(x)\}$	Conjunction
$A' = 1 - \mu_A(x)$	Negation
$A \rightarrow B = \min\{1, (1-\mu_A(x) + \mu_B(x)\}$	Implication
$A \text{ o } B = \min_x\{\mu_A(x), \mu_B(x)\}/x$	Composition

The Fuzzy propositions may contain quantifiers like "Very", "More or Less" . These Fuzzy quantifiers may be eliminated as

$\mu_{Very}(x) = \mu_A(x)^2$	Concentration
$\mu_{More\ or\ Less}(x) = \mu_A(x)^{1/2}$	Diffusion

Fuzzy reasoning is a drawing conclusion from Fuzzy propositions using fuzzy inference rules[14, 28]. Some of the Fuzzy inference rules are given bellow

R1: x is A
x and y are B

Y is $A \wedge B$

R2: x is A
x or y is B

---

y is $A \vee B$

R3: x and y are A
y and z are B

---

y and z are B

R4: x or y are A
y or z are B

---

x or z are B

R5: x is A
if x is A then y is B

y is A o (A→B)

3 Fuzzy Sets with Two Membership Functions

Zadeh[29] considered a single Fuzzy membership function to define the Fuzzy set to deal the uncertain information.

The proposition " x is A" is defined by

A= $\mu_A(x)$, Where A is Fuzzy Set and x Є X, $\mu A(x)$ is Fuzzy membership function.

The propositions " x is A" may represent the evidence with "Belief" and Disbelief" to deal the uncertain information.

Given some Universe of discourse X, the proposition " x is A" is defined by its two Fuzzy membership functions as

$\mu_A(x) = \{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$

or

$A = \{\mu_A^{Belief}(x), \mu_A^{Disbelief}(x)\}$

Where A is Fuzzy Set and x Є X and

$0 <= \mu_A^{Belief}(x) <=1$ and, $0 <= \mu_A^{Disbelief}(x) <=1$

Fuzzy set A = { $\mu_A^{Belief}(x_1)/x_1 + \mu_A^{Belief}(x_2)/x_2 + \ldots + \mu_A^{Belief}(x_n)/x_n$,
$\mu_A^{Disbelief}(x_1)/x_1 + \mu_A^{Disbelief}(x_2)/x_2 + \ldots + \mu_A^{Belief}(x_n)/x_n$, x_i Є X,
"+" is union

$\mu_A^{Belief}(x) + \mu_A^{Disbelief}(x) < 1$,
$\mu_A^{Belief}(x) + \mu_A^{Disbelief}(x) >1$
and $\mu_A^{Disbelief}(x) + \mu_A^{Disbelief}(x) = 1$
are interpreted as redundant, insufficient and sufficient Knowledge respectively.

For example,
Consider the Fuzzy proposition "x has Cold" and The Fuzzy set 'Cold" may be defined

Cold = { $0.8/x_1 + 0.6/x_2 + 0.4/x_3 + 0.6/x_4 +0.75/x_5$,
$0.4/x_1 + 0.5/x_2 + 0.5/x_3 + 0.4/x_4 +0.35/x_5$}

For instance "Rama has Cold" with Fuzziness 0.8

4 Fuzzy Logic and Fuzzy Reasoning with two Fuzzy Membership Functions

By considering Many-Valued Logic and Zadeh[] logical operations, the following operations may defined for Fuzzy Sets with two membership functions

Fuzzy logic is defined as combination of Fuzzy sets using logical operators. Some of the logical operations are given below

Suppose A, B, C are Fuzzy sets, and The operations on Fuzzy sets are given below

AVB=max { $\mu_A^{Belief}(x)$ V $\mu_A^{Belief}(x)$, $\mu_B^{Disbelief}(x)$ V $\mu_B^{Disbelief}(x)$} Disjunction

AΛB=min{ $\mu_A^{Belief}(x)$ Λ $\mu_A^{Belief}(x)$, $\mu_B^{Disbelief}(x)$ Λ $\mu_B^{Disbelief}(x)$ } Conjunction

A′= {1- $\mu_A^{Belief}(x)$, 1- $\mu_A^{Disbelief}(x)$ } Negation

A→B= {min (1, 1- $\mu_A^{Belief}(x)$ + $\mu_A^{Disbelief}(x)$, min (1, 1- $\mu_B^{Belief}(x)$ + $\mu_B^{Disbelief}(x)$} Implication

A o B= {$\min_x$ ($\mu_A^{Belief}(x)$, $\mu_A^{Belief}(x)$), $\min_x$($\mu_B^{Disbelief}(x)$, $\mu_B^{Disbelief}(x)$)}/x Composition

The Fuzzy propositions may contain quantifiers like "very", "more or less" . These Fuzzy quantifiers may be eliminated as

For the proposition "x is very A

$\mu_{Very\ A}(x)$ = { $\mu_A^{Belief}(x)^2$, $\mu_A^{Disbelief}(x)\mu_A(x)^2$ } Concentration

For proposition "x is more or less A"

$\mu_{More\ or\ Less\ A}(x)$ = ($\mu_A^{Belief}(x)^{1/2}$, $\mu_A^{Disbelief}(x)\mu_A(x)^{1/2}$ } Diffusion

In the propositions, the quantifiers are also particular about Belief and Disbelief. For the proposition "x is very A"

$\mu_{Very\ A}(x)$ = { $\mu_A^{Belief}(x)^2$, $\mu_A^{Disbelief}(x)\mu_A(x)$ }

For proposition "x is more or less A "

$\mu_{More\ or\ Less\ A}(x)$ = ($\mu_A^{Belief}(x)$, $\mu_A^{Disbelief}(x)\mu_A(x)^{1/2}$ }

Fuzzy reasoning[28] is drawing conclusions from Fuzzy propositions using fuzzy inference rules[1 Some of the Fuzzy inference rules are given below for the propositions with two membership functions

R1:	R2:
x is A	x is A
x and y are B	x or y is B
y is AΛB	y is AVB

AΛB = (min{$\mu_A^{Belief}(x)$, $\mu_A^{Belief}(x)$ }, min{$\mu_B^{Disbelief}(x)$, $\mu_B^{Disbelief}(x)$ })

$AVB = (\max\{\mu_A^{Belief}(x), \mu_A^{Belief}(x)\}, \quad \max\{\mu_B^{Disbelief}(x), \mu_B^{Disbelief}(x)\})$

R3: x and y are A	R4: x or y are A
y and z are B	y or z are B
y and z are B	x or z are B

$B = \{\mu_B^{Belief}(x), \quad \mu_B^{Disbelief}(x)\}$

R5: x is A
if x is A then y is B

y is A o (A→B)

$A \circ (A \rightarrow B) = (\min\{\mu_A^{Belief}(x), \min\{1, 1-\mu_A^{Belief}(x) + \mu_A^{Belief}(x)\}\}, \min\{\mu_B^{Disbelief}(x), \min\{1, 1-\mu_B^{Disbelief}(x), \mu_B^{Disbelief}(x)\}\})$

5 Fuzzy Modulations and Fuzzy Reasoning

. Fuzzy Modulations are a type of Knowledge representation for Fuzzy propositions [24].

The Fuzzy Modulation for the position “ x is A” is defined by

[A]R(x), where A is Fuzzy Set, R is relation and x Є X

Fuzzy reasoning is discussed for the Fuzzy sets with two membership functions Fuzzy modulations in the following

For instance,

“Rama has Cold” is modulated as
[Cold] Symptom(Rama)

The Fuzzy position “Rama has Headache” may be modulated as

[Headache] Symptom(Rama)

From the above propositions infer

Rama has Cold or Headache

This may be modulated as

[Cold V Headache Symptom(Rama)

For instance, consider the Fuzziness for Fuzzy Sets

Rama has cold
Cold = [0.6, 03]

Rama has Headache
Headache = {0.4, 0.5]

Rama has Cold or Headache
[Cold V Headache]Symptom(Rama)
[[0.6, 03]V{0.4, 0.5]]Symptom(Rama)
[[0.6, 0.5]]Symptom(Rama)
Rama has Cold or Headache with fuzziness [0.6, 0.5]

An Example of Fuzzy Reasoning with two membership functions is given bellow

Rama has Cold
If Rama has Cold Then Rama has Sneezing
If Rama has Cold Then Rama has Headache

The above Fuzzy facts may be modulated as

F1:[Cold] Symptom(Rama)
F2:If [Cold] Symptom(Rama) Then [Sneezing] Symptom(Rama)
or
F2: [(Cold → Sneezing] Symptom(Rama)
F3: If [Cold] Symptom(Rama) Then [Headache] Symptom(Rama)
Or
F3: [(Cold → Headache] Symptom(Rama)

From F1 and F2 infer using R5

F4: [Cold o (Cold → Sneezing] Symptom(Rama)

From F1 and F3 infer using R5

F5: [Cold o (Cold → Headache] Symptom(Rama)

If Rama has Sneezing Then Rama has Fever
If Rama has Headache Then Rama has Body pains

The above Fuzzy facts may be modulated as

F6: If [Sneezing] Symptom(Rama) Then [Fever] Symptom(Rama)
Or
F6: [Sneezing →Fever] Symptom(Rama)
F7: If [Headache] Symptom(Rama) Then [Body pains] Symptom(Rama)
Or
F7: [Headache →Body pains] Symptom(Rama)

From F4 and F6 infer

F8: [Cold o (Cold → Sneezing] o [Sneezing→fever] Symptom(Rama)

From F5 and F7 infer

F9:[Cold o (Cold→Headache]o [Headache→.Body pains] Symptom(Rama)

From F8 and F9 infer

F10: [Cold o (Cold → Sneezing] o [Sneezing → fever] [Sneezing] Symptom(Rama)
V
[Cold o (Cold → Headache] o [Headache →Body pains] [Sneezing V Body pains] Symptom(Rama)

For example,

Consider Fuzziness for the above propositions

Cold = [0.6, 03]
Fever = [0.4, 0.5]
Sneezing =[0.7, 0.2]
Headache= [0.4, 0.6]
Body pains = [0.7, 0.2]

F1:[Cold] Symptom(Rama)
[0.6, 03] Symptom(Rama)
F2: [Cold → Sneezing] Symptom(Rama)
[[0.6, 03] → [0.7, 0.2] Symptom(Rama)
[1, 0.9] Symptom(Rama)
F3: [Cold → Headache]Symptom(Rama)
[[0.6, 03]→ [0.4, 0.6]]Symptom(Rama)
[0.8,1] Symptom(Rama)

From F1 and F2 infer using R5

F4: [Cold o (Cold → Sneezing] Symptom(Rama)
[0.6, 03]o [1, 0.9] Symptom(Rama)
[0.6, 0.3]Symptom(Rama)

From F1 and F3 infer using R5

F5: [Cold o (Cold → Headache] Symptom(Rama)
[0.6, 03] o [0.8,1] Symptom(Rama)
[0.6, 0.3]Symptom(Rama)
F6: [Sneezing →Fever] Symptom(Rama)
F7: [Headache →Body pains] Symptom(Rama)

From F4 and F6 infer

F8: [Cold o (Cold → Sneezing] o [Sneezing→fever] Symptom(Rama)
[[0.6, 03]o ([0.6, 03]→ [[0.7, 0.2]) o [[0.7, 0.2]→ [0.4, 0.5]] Symptom(Rama)
[[0.6, 03]o [1, 0.9]] o [0.7, 1] Symptom(Rama)

[[0.6, 03]o [0.7, 1] Symptom(Rama)
[0.6, 03] Symptom(Rama)

From F5 and F7 infer

F9: [Cold o (Cold→Headache]o [Headache→.Body pains] Symptom(Rama)
[[0.6, 03]o ([0.6, 03]→ [0.4, 0.6]]o [[0.4, 0.6]→ [0.7, 0.2]] Symptom(Rama)
[[[0.6, 03]o [0.8,1]]o [[1,0.6] Symptom(Rama)
[0.6, 03]o [1,0.6] Symptom(Rama)
[0.6, 03] Symptom(Rama)

From F8 and F9 infer

F10: [Cold o (Cold → Sneezing] o [Sneezing → fever] [Sneezing] Symptom(Rama)
V
[Cold o (Cold → Headache] o [Headache →Body pains] [Sneezing V Body pains] Symptom(Rama)
[0.6, 03] Symptom(Rama) V[0.6, 03] Symptom(Rama)
[0.6, 0.3]V[0.6, 03] Symptom(Rama)
[0.6, 03] Symptom(Rama)

The inference is given by

Rama has Cold , Fever , Sneezing , Headache and Body pains with Fuzziness [0.6, 0.3], where Belief is 0.6 and Disbelief is 0.3

The above reasoning will more evidence with the two Fuzzy membership functions.

6 Fuzzy Certainty Factor

The Fuzzy Set with two membership function will give some more evidence then single Fuzzy membership function,

It is possible to define Fuzzy Set with single Fuzzy membership function for the Fuzzy Set with two membership functions

The Fuzzy Certainty Factor is defined by Fuzzy Set with single Fuzzy membership function with the difference of the two Fuzzy membership functions Belief and Disbelief.

$$\mu_A^{CF}(x) = \begin{cases} \mu_A^{Belief}(x) - \mu_A^{Disbelief}(x) & \mu_A^{Belief}(x) \geq \mu_A^{Disbelief}(x) \\ 0 & \mu_A^{Belief}(x) < \mu_A^{Disbelief}(x) \end{cases}$$

and

$\mu_A^{CF(x)}: X \rightarrow [0, 1]$, $x \in X$, where X is Universe of discourse.

Fuzzy Certainty Factor will compute the conflict of evidence in the Uncertain Information. If fuzzy certainty factor is less then or equal to zarero then the rule will be rejected.

The Fuzzy Logic and Fuzzy Reasoning for Fuzzy Certainty Factors can be studied similar lines as studied for Fuzzy Logic and Fuzzy Reasoning for fuzzy Sets with two Fuzzy membership functions.

7 Conclusions

The Fuzzy Set with two Fuzzy membership functions was defined. The operations on Fuzzy Sets with two Fuzzy membership functions were studied. The Fuzzy Logic and Fuzzy Reasoning were studied for the Fuzzy Sets with Two Fuzzy membership functions. The Fuzzy Modulations are discussed. The Fuzzy Logic and Fuzzy Reasoning were studied using Fuzzy modulations with the two Fuzzy membership functions. The Fuzzy Certainty Factor is defined by a single Fuzzy membership function to compute the conflict of evidence in the Uncertain Information. An example for Fuzzy Reasoning is given using Fuzzy Modulations for the Fuzzy Sets with two Fuzzy membership functions.

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